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Implementation of Kifilideen Trinomial Theorem based on Matrix Approach in Computation of the Power of Base of Tri-Digits Number

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The use of binomial theorem to expand trinomial expression of particular power n requires a bit by bit breaking down of the expansion until the final answer is obtained. This method had been seen not to be direct enough and involves repeated application up to the final result is achieved which makes the method to be unattractive. There is need to develop a direct and straight way in expanding trinomial expression of any power. This study implemented Kifilideen trinomial theorem based on matrix approach in computation of the power of base of tri – digits number. This method adopts Kif matrix approach in generating the power combination required in the evaluation. The use of Kifilideen trinomial theorem in expanding trinomial expression of positive power of n helps to eliminate humdrum nature of using binomial theorem in expanding such trinomial expression of positive power of n . The Kifilideen matrix approach in solving the tri – digits number of power n does not involve the repeated multiplication of figures unlike the use of long multiplication approach in solving such problem.

Keywords: Binomial theorem; Kifilideen matrix; Kifilideen trinomial theorem; Power of base number; Tri-digits number

1. Introduction

The binomial theorem is generally and commonly used to expand binomial expression of power of n [1], [2]. The theorem cannot be used to expand trinomial or multinomial expression of a particular power directly or in a straight forward way [3]. The process of doing that is cumbersome, complicated and tedious which can lead to loss of focus for large power of n [4], [5]. Trinomial expansion theorem is a theorem which helps to give long expression terms without involving any two or more terms of the expression enclosed with power [6]. [5] and [7] indicate that trinomial theorem has many applications in combinatorics, statistics, number theory and computing. Binomial and trinomial represent a special kind of polynomials [8]. Kifilideen trinomial theorem of positive power of n using matrix approach was developed by [9]. This Kifilideen trinomial theorem enables all the terms of the expansion of trinomial expression of positive power of n to be generated with easy and allow accurate prediction of the trinomial coefficients and any

term produce in the trinomial expansion of a very large power of n to be obtainable without any difficulty [10].

The Kifilideen trinomial theorem helps to expand trinomial expression of a given power in a periodicity and orderly manner which utilized straightforward procedure using matrix approach [9], [11]. The evaluation of tri – digits number of power of n using trinomial theorem is done by first developing the kif matrix of the tri – digits number of power of n then the computed terms of each row in the kif matrix is added together. The single column matrix obtained is further simplified to achieve the final answer. The kif matrix approach in solving the tri – digits number of power n does not involve the repeated multiplication of figures unlike the use of long multiplication approach in solving such problem. The used of binomial theorem to expand trinomial expression of particular power n required a bit by bit breaking down of the expansion until the final answer is obtained. This

method had been seen not direct enough and involve repeated application up to the final results is achieved. [10] revealed the need to develop a direct and straight way in expanding trinomial expression of any power of n . The use of Kifilideen trinomial theorem in expanding trinomial expression of positive power of n help to eliminate humdrum nature of using binomial theorem in expanding such trinomial expression of positive power of n . Focusing more on the utilization of trinomial theorem would help in revealing more of its hidden potential and capability to solve power of tri – digit number. As the saying, you can only know more about something when you continue having interaction with it [12].

[13] established kif matrix method of multiplication in the computation of power of base 11 and $abcd \times (11)^n$. It was indicated that this procedure can be extended to other power of base of di – digits number [14], [15]. [16] developed matrix approach in computation of power of base of di – digits number. This study implemented Kifilideen trinomial theorem based on matrix approach in computation of the power of base of tri – digits number.

2. Materials and Methods

2.1 Expansion of Power of Trinomial Expansion

The expansion of the power of trinomial expansion can be carried out directly using Kifilideen trinomial theorem.

2.2 Illustration of Expansion of Power of Trinomial Expansion using Kifilideen Trinomial Theorem

[a]. The power combination of a term in the Kif expansion of $\left[1 - \frac{x^3}{y^4} + x^2z\right]^{12}$ is 543. Determine the following using Kifilideen trinomial theorem:

- [i] the term of the power combination in the expansion
- [ii] state the group in which it belong to in the Kif matrix
- [iii] indicate the position of the power combination in that group
- [iv] total number of member in the group the power combination belong to
- [v] calculate the total number of groups produced by the expansion of the $\left[1 - \frac{x^3}{y^4} + x^2z\right]^{12}$ when arrange in the kif matrix

[vi] give the number of terms generated by the expansion.

Solution

$$[i] n = 5 + 4 + 3 = 12 \quad (1)$$

where n is the degree of the power of the trinomial expression

$$a = 3 \quad (2)$$

where a is the value of the third digit of the power combination or the value of the third digit of the column or group the term fall into

$$g = 3 + 1 = 4 \quad (3)$$

where g is group or column in which the term belong to

$$m = \frac{a}{2}[2n - a - 1] = \frac{3}{2}[2 \times 12 - 3 - 1] = 30 \quad (4)$$

Using Kifilideen power combination formula:

$$C_p = -90t + 81a + 90m - n90 \quad (5)$$

where $C_p, t, a,$ and n are power combination, t^{th} term of the kifilideen trinomial theorem where the power combination is found, the value of the third digit of the power combination or the value of the third digit of the column or group the term fall into and the power of the trinomial expression respectively. a and m are constant values for a particular group or column of the kif matrix.

$$543 = -90t + 81 \times 3 + 90 \times 30 + 1290 \quad (6)$$

$$t = 41^{\text{th}} \text{ term} \quad (7)$$

$$[ii] g = a + 1 = 3 + 1 = 4 \quad (8)$$

The power combination 543 belongs to group 4 of the kif matrix

[iii] To determine the position of the power combination 543 in the kif matrix the following formula is used:

$$R_{\text{member}} = -90r_n + 90 + F_{\text{member}} \quad (9)$$

where $R_{\text{member}}, F_{\text{member}}$ and r_n are the power combination of the required term to determine its position in the it belong group, the power combination of the first member in the stated group and the position of the power combination in the group its belong respectively.

F_{member} = first member in the group in which power combination 543 belongs to = 903

R_{member} = power combination of the required term to determine its position in the group = 543

$$543 = -90r_n + 90 + 903 \quad (10)$$

$$r_n = 5^{\text{th}} \text{ position} \quad (11)$$

This indicate that the power combination 543 is in the 5th position in group 4

[iv] Total number of member in the group the power combination belong to

$$g_n = n - a + 1 \tag{12}$$

where a, g_n and n are the power of the third digit of the any power combination in the group, total number of member in the particular group and the degree of the power of the trinomial expression respectively.

$$g_n = n - a + 1 = 12 - 3 + 1 = 10 \tag{13}$$

[v] The total number of groups that can be generated by the expansion $\left[1 - \frac{x^3}{y^4} + x^2z\right]^{12}$ is given as

$$g_{max} - n + 1 = 12 + 1 = 13 \tag{14}$$

where g_{max} is the maximum number of groups that can be generated by the expansion

[vi] Number of terms generated by the trinomial expansion
$$= \left[\frac{(n+1)(n+2)}{2} \right] = \frac{(12+1)(12+2)}{2} = 91$$
 terms (15)

where n is power of the trinomial expansion

[b] Indicate the power combination of the 38th term of the kif expansion of $\left[4 - \frac{x^2}{z^5} + x^3y^4\right]^{10}$. Thus, determine the value of term of the power combination.

Solution

From Kifilideen general group formula, we have:

$$t = \frac{g}{2}(2n - g + 3) \tag{16}$$

where t, g and n are the t^{th} term of the kifilideen trinomial theorem where the power combination is found and group or column in which the term belong to and the power of the trinomial expression respectively.

From the question, $n = 10$ (17)

$$38 = \frac{g}{2}(2 \times 10 - g + 3) \tag{18}$$

Using quadratic formula,

$$g = \frac{23 \pm \sqrt{23^2 - 4 \times 1 \times 76}}{2 \times 1} = \frac{23 \pm \sqrt{225}}{2} = \frac{23 \pm 15}{2} = 19 \text{ or } 4 \tag{19}$$

$$g_{max} = n + 1 = 10 + 1 = 11 \tag{20}$$

where g_{max} is the maximum number of groups that can be generated by the expansion

Therefore the 38th term belongs to group 4 that is
$$g = 4 \tag{21}$$

$$a = g - 1 = 4 - 1 = 3 \tag{22}$$

$$m = \frac{a}{2}(2n - a - 1) = \frac{3}{2}(2 \times 10 - 3 - 1) = 24 \tag{23}$$

From Kifilideen general power combination formula,

$$C_p = -90t + 81a + 90m + n90 = -90 \times 38 + 81 \times 3 + 90 \times 24 + 1090 = 073 \tag{24}$$

The power combination of the 38th term = 073 (25)

The term of the 38th =
$${}_{0,7,3}C [4]^0 \left[-\frac{x^2}{z^5}\right]^7 [x^3y^4]^3 = -\frac{10!}{0!7!3!} x^{23} y^{12} z^{-35} = -120x^{23} y^{12} z^{-35} \tag{26}$$

The term of the 38th = $-120x^{23} y^{12} z^{-35}$ (27)

[c] Determine the coefficient of $x^{-7}y^6z^{12}$ in the expansion of $\left[\frac{x^2}{3y^3} + x^2y^2z - \frac{yz^3}{x^5}\right]^7$. Hence, find the power combination of the term that produce the coefficient.

Solution

The general expression of any of the terms of the kif trinomial expansion of the expression $\left[\frac{x^2}{3y^3} + x^2y^2z - \frac{yz^3}{x^5}\right]^7$ is given as:

$${}_{a,b,c}C \left[\frac{x^2}{3y^3}\right]^a [x^2y^2z]^b \left[-\frac{yz^3}{x^5}\right]^c \tag{28}$$

$${}_{a,b,c}C [-1]^c [3^{-1}]^a [x]^{2a+2b-5c} [y]^{-3a+2b+c} [z]^{b+3c} \tag{29}$$

Let the term with $x^{-7}y^6z^{12}$ gives $kx^{-7}y^6z^{12}$ (30)

where, k is the coefficient of the term

So comparing (30) with $kx^{-7}y^6z^{12}$ we have,
$$2a + 2b - 5c = -7 \tag{31}$$

$$-3a + 2b + c = 6 \tag{32}$$

$$b + 3c = 12 \tag{33}$$

Using Crammer's rule, we

$$a = 1, b = 3 \text{ and } c = 3 \tag{34}$$

Therefore,

$$k = {}_{a,b,c}C [-1]^c [3^{-1}]^a = {}_{1,3,3}C [-1]^3 [3^{-1}]^1 = -\frac{7!}{1!3!3!} \times \frac{1}{3} = -\frac{140}{3} \tag{35}$$

$$k = -\frac{140}{3} \tag{36}$$

[d] A term in the expansion of $\left[135 - \frac{k}{6x^2} - 5x^5\right]^6$ yield $-60,000 x^4$. Find the possible value of k .

Solution

The general expression of any of the terms of the kif trinomial expansion of the expression

$$\left[135 - \frac{k}{6x^2} - 5x^5\right]^6 \text{ is given as: } {}_{a,b,c}C [135]^a \left[-\frac{k}{6x^2}\right]^b [-5x^5]^c \tag{37}$$

$${}_{a,b,c}^6C[-1]^b [-1]^c [135]^a [6^{-1}]^b [5]^c [k]^b [x]^{-2b+5c} \quad (38)$$

Comparing (38) with $-60,000 x^4$ we have: (39)

$$-2b + 5c = 4 \quad (40)$$

Furthermore,

$$a + b + c = 6 \quad (41)$$

Note, since b and c are parts of the power combination, the possible value of b and c are 0, 1, 2, 3, 4, 5 and 6.

$$\text{For } b = 0 \quad (42)$$

$$-2b + 5c = 4 \quad (43)$$

$$c = \frac{4}{5} \quad (44)$$

Component of power combination cannot be fraction so b can be equal to 0

$$\text{For } b = 1 \quad (45)$$

$$-2b + 5c = 4 \quad (46)$$

$$c = \frac{6}{5} \quad (47)$$

$$\text{So, } b \neq 1 \quad (48)$$

$$\text{For } b = 2 \quad (49)$$

$$-2b + 5c = 4 \quad (50)$$

$$c = \frac{8}{5} \quad (51)$$

$$\text{So, } b \neq 2 \quad (52)$$

$$\text{For } b = 3 \quad (53)$$

$$-2b + 5c = 4 \quad (54)$$

$$c = \frac{10}{5} = 2 \quad (55)$$

$$a + b + c = 6 \quad (56)$$

$$a + 3 + 2 = 6 \quad (57)$$

$$a = 1 \quad (58)$$

So, for $b = 3$, the power combination is 132 (59)

$$\text{For } b = 4 \quad (60)$$

$$-2b + 5c = 4 \quad (61)$$

$$c = \frac{12}{5} \quad (62)$$

$$\text{So, } b \neq 4 \quad (63)$$

$$\text{For } b = 5 \quad (64)$$

$$-2b + 5c = 4 \quad (65)$$

$$c = \frac{14}{5} \quad (66)$$

$$\text{So, } b \neq 5 \quad (67)$$

$$\text{For } b = 6 \quad (68)$$

$$-2b + 5c = 4 \quad (69)$$

$$c = \frac{14}{5} \quad (70)$$

$$\text{So, } b \neq 6 \quad (71)$$

The power combinations that would yield the simplified term of expression $-60,000 x^4$ is 132.

From,

$${}_{a,b,c}^6C[-1]^b [-1]^c [135]^a [6^{-1}]^b [5]^c [k]^b [x]^{-2b+5c} = -60,000 x^4 \quad (72)$$

$${}_{1,3,2}^6C[-1]^3 [-1]^2 [135]^1 [6^{-1}]^3 [5]^2 [k]^3 [x]^{-2 \times 3 + 5 \times 2} = -60,000 x^4 \quad (73)$$

$$-60 \times [135]^1 \times [6^{-1}]^3 \times [5]^2 [k]^3 [x]^4 = -60,000 x^4 \quad (74)$$

$$\frac{-60 \times 135 \times 25}{216} [k]^3 = -60,000 \quad (75)$$

$$[k]^3 = 64 \quad (76)$$

$$k = [64]^{\frac{1}{3}} \quad (77)$$

$$k = 4 \quad (78)$$

[e] Expand the expression $\left[1 - \frac{4}{x} + \frac{x^3}{3}\right]^4$ using Kifilideen theorem theorem

Solution

The combination power for the expansion of the trinomial expression of power 4 in ascending order of terms using kif trinomial theorem arrangement are 400, 310, 220, 130, 040, 301, 211, 121, 031, 202, 112, 022, 103, 013 and 004.

$$\begin{aligned}
 \left[1 - \frac{4}{x} + \frac{x^3}{3}\right]^4 &= {}_{4,0,0}^4C[1]^4 \left[-\frac{4}{x}\right]^0 \left[\frac{x^3}{3}\right]^0 + \\
 &{}_{3,1,0}^4C[1]^3 \left[-\frac{4}{x}\right]^1 \left[\frac{x^3}{3}\right]^0 + {}_{2,2,0}^4C[1]^2 \left[-\frac{4}{x}\right]^2 \left[\frac{x^3}{3}\right]^0 + \\
 \left[1 - \frac{4}{x} + \frac{x^3}{3}\right]^4 &= {}_{4,0,0}^4C[1]^4 \left[-\frac{4}{x}\right]^0 \left[\frac{x^3}{3}\right]^0 + {}_{3,1,0}^4C[1]^3 \left[-\frac{4}{x}\right]^1 \left[\frac{x^3}{3}\right]^0 + {}_{2,2,0}^4C[1]^2 \left[-\frac{4}{x}\right]^2 \left[\frac{x^3}{3}\right]^0 + \\
 &{}_{1,3,0}^4C[1]^1 \left[-\frac{4}{x}\right]^3 \left[\frac{x^3}{3}\right]^0 + {}_{0,4,0}^4C[1]^0 \left[-\frac{4}{x}\right]^4 \left[\frac{x^3}{3}\right]^0 + {}_{3,0,1}^4C[1]^3 \left[-\frac{4}{x}\right]^0 \left[\frac{x^3}{3}\right]^1 + {}_{2,1,1}^4C[1]^2 \left[-\frac{4}{x}\right]^1 \left[\frac{x^3}{3}\right]^1 \\
 &+ {}_{1,2,1}^4C[1]^1 \left[-\frac{4}{x}\right]^2 \left[\frac{x^3}{3}\right]^1 + {}_{0,3,1}^4C[1]^0 \left[-\frac{4}{x}\right]^3 \left[\frac{x^3}{3}\right]^1 + {}_{2,0,2}^4C[1]^2 \left[-\frac{4}{x}\right]^0 \left[\frac{x^3}{3}\right]^2 + {}_{1,1,2}^4C[1]^1 \left[-\frac{4}{x}\right]^1 \left[\frac{x^3}{3}\right]^2 \\
 &+ {}_{0,2,2}^4C[1]^0 \left[-\frac{4}{x}\right]^2 \left[\frac{x^3}{3}\right]^2 + {}_{1,0,3}^4C[1]^1 \left[-\frac{4}{x}\right]^0 \left[\frac{x^3}{3}\right]^3 + {}_{0,1,3}^4C[1]^0 \left[-\frac{4}{x}\right]^1 \left[\frac{x^3}{3}\right]^3 + {}_{0,0,4}^4C[1]^0 \left[-\frac{4}{x}\right]^0 \left[\frac{x^3}{3}\right]^4
 \end{aligned}$$

(79)

$$\left[1 - \frac{4}{x} + \frac{x^3}{3}\right]^4 = 1 - \frac{16}{x} + \frac{96}{x^2} - \frac{256}{x^3} + \frac{256}{x^4} + \frac{4x^3}{3} - 16x^2 + 64x - \frac{256}{3} + \frac{2x^6}{3} - \frac{16x^5}{3} + \frac{32x^4}{3} + \frac{4x^9}{3} - \frac{16x^8}{3} + \frac{x^{12}}{3} \quad (80)$$

[f] Expand in the power of x $[3 - x + 2x^2]^5$ using kif trinomial theorem. If $3 - x + 2x^2 = 2.895$ hence evaluate $[2.895]^5$.

[f] Expand in the power of x $[3 - x + 2x^2]^5$ using kif trinomial theorem. If $3 - x + 2x^2 = 2.895$ hence evaluate $[2.895]^5$.

Solution

The combination power for the expansion of the trinomial expression of power 5 in ascending order of terms are 500, 410, 320, 230, 140, 050, 401, 311, 221, 131, 041, 302, 212, 122, 032, 203, 113, 023, 104, 014 and 005.

$$\begin{aligned}
 [3 - x + 2x^2]^5 &= {}_{5,0,0}^5C(3)^5 (-x)^0 (2x^2)^0 + {}_{4,1,0}^5C(3)^4 (-x)^1 (2x^2)^0 + {}_{3,2,0}^5C(3)^3 (-x)^2 (2x^2)^0 + \\
 &{}_{2,3,0}^5C(3)^2 (-x)^3 (2x^2)^0 + {}_{1,4,0}^5C(3)^1 (-x)^4 (2x^2)^0 + {}_{0,5,0}^5C(3)^0 (-x)^5 (2x^2)^0 + {}_{4,0,1}^5C(3)^4 (-x)^0 (2x^2)^1 \\
 &+ {}_{3,1,1}^5C(3)^3 (-x)^1 (2x^2)^1 + {}_{2,2,1}^5C(3)^2 (-x)^2 (2x^2)^1 + {}_{1,3,1}^5C(3)^1 (-x)^3 (2x^2)^1 + \\
 &{}_{0,4,1}^5C(3)^0 (-x)^4 (2x^2)^1 + {}_{3,0,2}^5C(3)^3 (-x)^0 (2x^2)^2 + {}_{2,1,2}^5C(3)^2 (-x)^1 (2x^2)^2 + {}_{1,2,2}^5C(3)^1 (-x)^2 (2x^2)^2 \\
 &+ {}_{0,3,2}^5C(3)^0 (-x)^3 (2x^2)^2 + {}_{2,0,3}^5C(3)^2 (-x)^0 (2x^2)^3 + {}_{1,1,3}^5C(3)^1 (-x)^1 (2x^2)^3 + \\
 &{}_{0,2,3}^5C(3)^0 (-x)^2 (2x^2)^3 + {}_{1,0,4}^5C(3)^1 (-x)^0 (2x^2)^4 + {}_{0,1,4}^5C(3)^0 (-x)^1 (2x^2)^4 + \\
 &{}_{0,0,5}^5C(3)^0 (-x)^0 (2x^2)^5
 \end{aligned}$$

(81)

$$\begin{aligned}
 [3 - x + 2x^2]^5 &= 243 \\
 &-405x + 270x^2 - 90x^3 + 15x^4 - x^5 + 810x^2 - 1080x^3 + 540x^4 - 120x^5 + 10x^6 + 1080x^4 - 1080x^5 + \\
 &360x^6 - 40x^7 + 720x^6 - 480x^7 + 80x^8 + 240x^8 - 80x^9 + 32x^{10}
 \end{aligned}$$

(82)

$$\begin{aligned}
 [3 - x + 2x^2]^5 &= 243 - [405]x + [270 + 810]x^2 + [-90 - 1080]x^3 + [15 + 540 + 1080]x^4 \\
 &+ [-1 - 120 - 1080]x^5 + [10 + 360 + 720]x^6 + [-40 - 480]x^7 + [80 + 240]x^8 + [-80]x^9 + [32]x^{10} \\
 [3 - x + 2x^2]^5 &= 243 - 405x + 1080x^2 - 1170x^3 + 1635x^4 - 1201x^5 + 1090x^6 - 520x^7 + 320x^8 - \\
 &80x^9 + 32x^{10}
 \end{aligned}$$

(84)

[ii] $3 - x + 2x^2 = 2.895 = 3 - 0.105$ (85)

So, $-x + 2x^2 = -0.105$ (86)

$2x^2 - x + 0.105 = 0$ (87)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (88)$$

$a = 2, b = -1$ and $c = 0.105$ (89)

$$x = \frac{-[-1] \pm \sqrt{[-1]^2 - 4 \times 2 \times 0.105}}{2 \times 2} \quad (90)$$

$$x = \frac{1 \pm 0.4}{4} \quad (91)$$

$$x = 0.35 \text{ or } 0.15 \quad (92)$$

from,

$$[3 - x + 2x^2]^5 = 243 + 405x + 1080x^2 - 1170x^3 + 1635x^4 - 1201x^5 + 1090x^6 - 520x^7 + 320x^8 - 80x^9 + 32x^{10}$$

(93)

$$[2.895]^5 = 243 - 405 [0.35] + 1080[0.35]^2 - 1170[0.35]^3 + 1635[0.35]^4 - 1201[0.35]^5 + 1090[0.35]^6 - 520[0.35]^7 + 320[0.35]^8 - 80[0.35]^9 + 32[0.35]^{10}$$

(94)

$$[2.895]^5 = 203.3493742 \quad (95)$$

[g] In the expansion of $[3 - x + ax^2]^5$ and in the simplification of the terms generate, one of the terms has an expression of $1080x^2$. Determine the possible value of a.

Solution

The general expression of any of the terms of the kif trinomial expansion of the expression

$$[3 - x + ax^2]^5 \text{ is}$$

$${}_{r,s,t}^5C (3)^r (-x)^s (ax^2)^t \quad (96)$$

$${}_{r,s,t}^5C (3)^r (-1)^s (x)^s (a)^t (x^2)^t \quad (97)$$

$${}_{r,s,t}^5C (3)^r (-1)^s (a)^t (x)^{s+2t} \quad (98)$$

Comparing (98) with $1080x^2$, we have

$$s + 2t = 2 \quad (99)$$

$$\text{Also, } r + s + t = 5 \quad (100)$$

Note since s and t are parts of the power combination, the possible values of s and t are 0, 1, 2, 3, 4, 5.

For $t = 0$,

$$s + 2t = 2 \quad (101)$$

$$s = 2 \quad (102)$$

$$r + s + t = 5 \quad (103)$$

$$r + 2 + 0 = 5 \quad (104)$$

$$r = 3 \quad (105)$$

So, for $t = 0$ the power combination is 320.

For $t = 1$,

$$s + 2t = 2 \quad (106)$$

$$s = 0 \quad (107)$$

$$r + s + t = 5 \quad (108)$$

$$r + 0 + 1 = 5 \quad (109)$$

$$r = 4 \quad (110)$$

Moreover, $t \neq 2$ or have higher value since it would make the value of s to be negative in equation (99). So, the power combinations that would generate the simplified term of expression $1080x^2$ are 320 and 401. Both 320 and 401 are in row three in the kif matrix for power of 5.

So, for $t = 1$ the power combination is 401.

$${}_{3,2,0}^5C (3)^r (-1)^s (a)^t (x)^{s+2t} + {}_{4,0,1}^5C (3)^r (-1)^s (a)^t (x)^{s+2t} = 1080x^2 \quad (111)$$

$${}_{3,2,0}^5C (3)^3 (-1)^2 (a)^0 (x)^{2+2 \times 0} + {}_{4,0,1}^5C (3)^4 (-1)^0 (a)^1 (x)^{0+2 \times 1} = 1080x^2 \quad (112)$$

$$270 (x)^2 + 405 (a)^1 (x)^2 = 1080x^2 \quad (113)$$

$$405a = 1080 - 270 = 810 \quad (114)$$

$$a = 2 \quad (115)$$

[h] Find the coefficient of x^3 in the expansion of $[2 + x + 3x^2]^6$

Solution

The general expression of any of the terms of the kif trinomial expansion of the expression $[2 + x + 3x^2]^6$ is

$${}_{r,s,t}C (2)^r (x)^s (3x^2)^t \tag{116}$$

$${}_{r,s,t}C (2)^r (x)^s (3)^t (x^2)^t \tag{117}$$

$${}_{r,s,t}C (2)^r (3)^t (x)^{s+2t} \tag{118}$$

Let the terms with x^3 be cx^3

Comparing (118) with cx^3 , we have

$$s + 2t = 3 \tag{119}$$

Also, $r + s + t = 6$ (120)

Note since s and t are parts of the power combination, the possible values of s and t are 0, 1, 2, 3, 4, 5, 6.

For $t = 0$,
 $s + 2t = 3$ (121)

$$s = 3 \tag{122}$$

$$r + s + t = 6 \tag{123}$$

$$r + 3 + 0 = 6 \tag{124}$$

$$r = 3 \tag{125}$$

So, for $t = 0$ the power combination is 330.

For $t = 1$,
 $s + 2t = 3$ (126)

$$s = 1 \tag{127}$$

$$r + s + t = 6 \tag{128}$$

$$r + 1 + 1 = 6 \tag{129}$$

$$r = 4 \tag{130}$$

So, for $t = 1$ the power combination is 411.

For $t = 2$,
 $s + 2t = 3$ (131)

$$s = 1 \tag{132}$$

$$r + s + t = 6 \tag{133}$$

$$r + 1 + 1 = 6 \tag{134}$$

$$r = 4 \tag{135}$$

Moreover, $t \neq 2$ or have higher value since it would make the value of s to be negative in equation (132). So, the power combinations that would generate the simplified term of expression cx^3 are 330 and 411. Both 330 and 411 power combinations present in row 4 in the kif matrix for power of 6.

$${}_{3,3,0}C (2)^r (3)^t (x)^{s+2t} + {}_{4,1,1}C (2)^r (3)^t (x)^{s+2t} = cx^3 \tag{136}$$

$${}_{3,3,0}C (2)^3 (3)^0 (x)^{3+2 \times 0} + {}_{4,1,1}C (2)^4 (3)^1 (x)^{1+2 \times 1} = cx^3 \tag{137}$$

$$160 (x)^3 + 1440 (x)^3 = 1600 x^3 \tag{138}$$

The coefficient of x^3 in the expansion of $[2 + x + 3x^2]^6$ is 1600

[i] Find the coefficient of x^{-9} in the expansion of $\left[1 + \frac{x^2}{y^3} + \frac{yz^2}{x^3}\right]^8$

Solution

The general expression of any of the terms of the kif trinomial expansion of the

expression $\left[1 + \frac{x^2}{y^3} + \frac{yz^2}{x^3}\right]^8$ is

$${}_{r,s,t}C (1)^r \left(\frac{x^2}{y^3}\right)^s \left(\frac{yz^2}{x^3}\right)^t \tag{139}$$

$${}_{r,s,t}C (1)^r (x^2)^s (y^{-3})^s (y)^t (z^2)^t (x^{-3})^t \tag{140}$$

$${}_{r,s,t}C (1)^r (x)^{2s-3t} (y)^{-3s+t} (z)^{2t} \tag{141}$$

Let the terms with x^{-9} be ax^{-9}

Comparing (141) with ax^{-9} , we have

$$2s - 3t = -9 \tag{142}$$

$$\text{Also, } r + s + t = 8 \quad (143)$$

Note since s and t are parts of the power combination, the possible values of s and t are 0, 1, 2, 3, 4, 5, 6, 7, 8.

For $t = 0, 4, 6$ and 8, the values of s are fractions which is not possible.

For $t = 1$ and 2 the values of s are negative which is not possible.

So, for $t = 3$

$$2s - 3t = -9 \quad (144)$$

$$s = 0 \quad (145)$$

$$r + s + t = 8 \quad (146)$$

$$r + 0 + 3 = 8 \quad (147)$$

$$r = 5 \quad (148)$$

Therefore, for $t = 3$ the power combination is 503

For $t = 5$,

$$2s - 3t = -9 \quad (149)$$

$$s = 3 \quad (150)$$

$$r + s + t = 8 \quad (151)$$

$$r + 3 + 5 = 8 \quad (152)$$

$$r = 0 \quad (153)$$

So, for $t = 5$ the power combination is 035.

For $t = 7$,

$$2s - 3t = -9 \quad (154)$$

$$s = 6 \quad (155)$$

$$r + s + t = 8 \quad (156)$$

$$r + 6 + 7 = 8 \quad (157)$$

$$r = -5 \quad (158)$$

Moreover, $r \neq -5$ since part of the power combination cannot be negative. So, the power combinations that would generate the simplified term of expression ax^{-9} are 035 and 503.

$${}_{0,3,5}^8C (1)^r (x)^{2s-3t} (y)^{-3s+t} (z)^{2t} + {}_{5,0,3}^8C (1)^r (x)^{2s-3t} (y)^{-3s+t} (z)^{2t} = ax^{-9} \quad (159)$$

$${}_{0,3,5}^8C (1)^0 (x)^{2 \times 3 - 3 \times 5} (y)^{-3 \times 3 + 5} (z)^{2 \times 5} + {}_{5,0,3}^6C (1)^5 (x)^{2 \times 0 - 3 \times 3} (y)^{-3 \times 0 + 3} (z)^{2 \times 3} = ax^{-9} \quad (160)$$

$$56 (y)^{-4} (z)^{10} (x)^{-9} + 56 (y)^3 (z)^6 (x)^{-9} = ax^{-9} \quad (161)$$

$$[56 (y)^{-4} (z)^{10} + 56 (y)^3 (z)^6] x^{-9} = ax^{-9} \quad (162)$$

The coefficient of x^{-9} in the expansion of $\left[1 + \frac{x^2}{y^3} + \frac{yz^2}{x^3}\right]^8$ is $\left[\frac{56z^{10}}{y^4} + 56y^3z^6\right]$ or $\left[\frac{56z^{10} + 56y^7z^6}{y^4}\right]$

2.3 Computation of Power of Base of Three Digits Number using Kif Matrix Method

The computation of power of base of tri-digits number using kif matrix method provides an easy way procedure. Tri-digit number is a number with three digits. Tri-digit number can also be referred to as three digits number. This method prevents repeated method of multiplication use in long multiplication. It involves the arrangement of the power combination of the terms generated by the kif expansion of trinomial expression of power n in a particular row in rows and columns when the three digits number were added and raise to the power given. All the terms for each row in the kif

matrix produced is added together. The single column matrix achieved is further simplified to obtain the final answer.

2.3.1 Computation of Base of Three Digits Number of Power of 2

In the expansion of $(xyz)^2$ the kif matrix of the expansion of trinomial expression of power of 2 is obtained as follow. Each term of the matrix is generated and the all the terms for each row is added together.

$$\begin{bmatrix} 200 \\ 110 \\ 020 + 101 \\ \quad \quad 011 \\ \quad \quad \quad \quad 002 \end{bmatrix}$$

$$(xyz)^2 = \begin{bmatrix} {}_{2,0,0}^2C x^2y^0z^0 \\ {}_{2,1,1}^2C x^2y^1z^1 \\ {}_{0,2,0}^2C x^0y^2z^0 + {}_{1,0,1}^2C x^1y^0z^1 \\ {}_{0,1,1}^2C x^0y^1z^1 \\ {}_{0,0,2}^2C x^0y^0z^2 \end{bmatrix} \quad (164)$$

The single column matrix obtained is further simplified to achieve the final answer.

2.4 Illustration on the Computation of Base of Three Digits Number of Power of 2

Expand the following using kifilideen matrix approach

[i] $(234)^2$ [ii] $(875)^2$

Solution

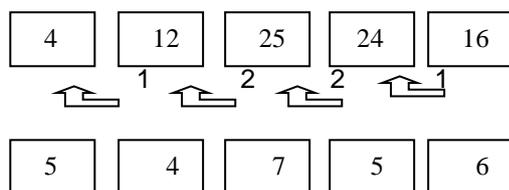
[i] $(234)^2$

The Kifilideen matrix to solve the expression is given has

$$\begin{bmatrix} 200 \\ 110 \\ 020 + 101 \\ 011 \\ 002 \end{bmatrix}$$

So, we have

$$(234)^2 = \begin{bmatrix} {}_{2,0,0}^2C 2^23^04^0 \\ {}_{1,1,0}^2C 2^13^14^0 \\ {}_{0,2,0}^2C 2^03^24^0 + {}_{1,0,1}^2C 2^13^04^1 \\ {}_{0,1,1}^2C 2^03^14^1 \\ {}_{0,0,2}^2C 2^03^04^2 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 9+16 \\ 24 \\ 16 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 25 \\ 24 \\ 16 \end{bmatrix}$$



$(234)^2 = 54356$ or 5.4756×10^4 (165)

[ii] $(875)^2$

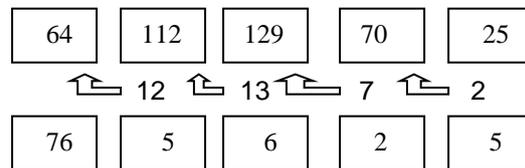
The Kifilideen matrix to solve the expression is given has

$$\begin{bmatrix} 200 \\ 110 \\ 020 + 101 \\ 011 \\ 002 \end{bmatrix}$$

So, we have

$$(875)^2 = \begin{bmatrix} {}_{2,0,0}^2C 8^27^05^0 \\ {}_{1,1,0}^2C 8^17^15^0 \\ {}_{0,2,0}^2C 8^07^25^0 + {}_{1,0,1}^2C 8^17^05^1 \\ {}_{0,1,1}^2C 8^07^15^1 \\ {}_{0,0,2}^2C 8^07^05^2 \end{bmatrix} \quad (166)$$

$$(875)^2 = \begin{bmatrix} 64 \\ 112 \\ 49 + 80 \\ 70 \\ 25 \end{bmatrix} = \begin{bmatrix} 64 \\ 112 \\ 129 \\ 70 \\ 25 \end{bmatrix} \quad (167)$$



$(875)^2 = 765625$ or 7.65625×10^5 (168)

2.5 Computation of Base of Three Digits Number of Power of Base 5

In the expansion of $(xyz)^5$ the kifilideen matrix of the expansion of trinomial expression of power of 5 is obtained as follow. Each term of the matrix is generated and the all the terms for each row is added together.

$$\begin{bmatrix} 500 \\ 410 \\ 320 + 401 \\ 230 + 311 \\ 140 + 221 + 302 \\ 050 + 131 + 212 \\ 041 + 122 + 203 \\ 032 + 113 \\ 023 + 104 \\ 014 \\ 005 \end{bmatrix} \quad (169)$$

$$(xyz)^5 = \begin{bmatrix} {}_{5,0,0}^5C x^5y^0z^0 \\ {}_{4,1,0}^5C x^4y^1z^0 \\ {}_{3,2,0}^5C x^3y^2z^0 + {}_{4,0,1}^5C x^4y^0z^1 \\ {}_{2,3,0}^5C x^2y^3z^0 + {}_{1,0,1}^5C x^3y^1z^1 \\ {}_{1,4,0}^5C x^0y^1z^4 + {}_{0,1,4}^5C x^0y^1z^4 + {}_{0,1,4}^5C x^0y^1z^4 \\ {}_{0,5,0}^5C x^0y^1z^4 + {}_{0,1,4}^5C x^0y^1z^4 + {}_{0,1,4}^5C x^0y^1z^4 \\ {}_{0,2,3}^5C x^0y^2z^3 + {}_{1,0,4}^5C x^1y^0z^4 \\ {}_{0,1,4}^5C x^0y^1z^4 \\ {}_{0,0,5}^5C x^0y^0z^5 \end{bmatrix} \quad (170)$$

The single column matrix obtained is further simplified to achieve the final answer.

2.5.1 Illustration on the Computation of Base of Three Digits Number of Power of 5 and 4

The Kifilideen matrix to solve the expression is given has

$$\left[\begin{array}{r} 500 \\ 410 \\ 320 + 401 \\ 230 + 311 \\ 140 + 221 + 302 \\ 050 + 131 + 212 \\ \quad 041 + 122 + 203 \\ \quad \quad 032 + 113 \\ \quad \quad \quad 023 + 104 \\ \quad \quad \quad \quad 014 \\ \quad \quad \quad \quad \quad 005 \end{array} \right] \quad (171)$$

[a] Evaluate the following using kif matrix approach

[i] $(420)^5$ [ii] $(321)^4$

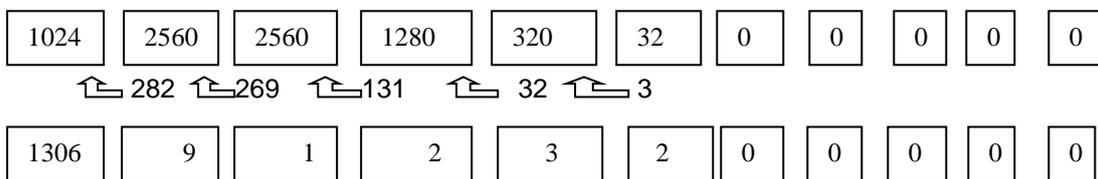
Solution

[i] $(420)^5$

So, we have

$$(420)^5 = \left[\begin{array}{r} {}_{5,0,0}C 4^5 2^0 0^0 \\ {}_{4,1,0}C 4^4 2^1 0^0 \\ {}_{3,2,0}C 4^3 2^2 0^0 + {}_{4,0,1}C 4^4 2^0 0^1 \\ {}_{2,3,0}C 4^2 2^3 0^0 + {}_{3,1,1}C 4^3 2^1 0^1 \\ {}_{1,4,0}C 4^1 2^4 0^0 + {}_{2,2,1}C 4^2 2^2 0^1 + {}_{3,0,2}C 4^3 2^0 0^2 \\ {}_{0,5,0}C 4^0 2^5 0^0 + {}_{1,3,1}C 4^1 2^3 0^1 + {}_{2,1,2}C 4^2 2^1 0^2 \\ {}_{0,4,1}C 4^0 2^4 0^1 + {}_{1,2,2}C 4^1 2^2 0^2 + {}_{2,0,3}C 4^2 2^0 0^3 \\ {}_{0,3,2}C 4^0 2^3 0^2 + {}_{1,1,3}C 4^1 2^1 0^3 \\ {}_{0,2,3}C 4^0 2^2 0^3 + {}_{1,0,4}C 4^1 2^0 0^4 \\ {}_{0,1,4}C 4^0 2^1 0^4 \\ {}_{0,0,5}C 4^0 2^0 0^5 \end{array} \right] \quad (172)$$

$$(420)^5 = \left[\begin{array}{r} 1024 \\ 2560 \\ 2560 \\ 1280 \\ 320 \\ 32 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right] \quad (173)$$



$(420)^5 = 13069123200000$ or $1.30691232 \times 10^{13}$ (174)

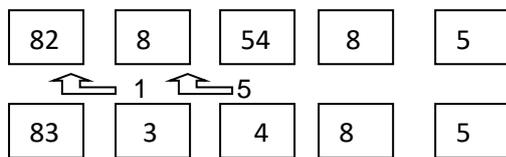
[ii] $(321)^4$

The Kifilideen matrix to solve the expression is given has

So, we have

$$[902]^2 + [141]^2 = \begin{bmatrix} {}_{2,0,0}^2C 9^2 0^0 2^0 \\ {}_{1,1,0}^2C 9^1 0^1 2^0 \\ {}_{0,2,0}^2C 9^0 0^2 2^0 + {}_{1,0,1}^2C 9^1 0^0 2^1 \\ {}_{0,1,1}^2C 9^0 0^1 2^1 \\ {}_{0,0,2}^2C 9^0 0^0 2^2 \end{bmatrix} + \begin{bmatrix} {}_{2,0,0}^2C 1^2 4^0 1^0 \\ {}_{1,1,0}^2C 1^1 4^1 1^0 \\ {}_{0,2,0}^2C 1^0 4^2 1^0 + {}_{1,0,1}^2C 1^1 4^0 1^1 \\ {}_{0,1,1}^2C 1^0 4^1 1^1 \\ {}_{0,0,2}^2C 1^0 4^0 1^2 \end{bmatrix} \quad (179)$$

$$[902]^2 + [141]^2 = \begin{bmatrix} 81 \\ 0 \\ 0 + 36 \\ 0 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 8 \\ 16 + 2 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 82 \\ 8 \\ 54 \\ 8 \\ 5 \end{bmatrix} \quad (180)$$



$$[902]^2 + [141]^2 = 851545 \text{ or } 8.51545 \times 10^5 \quad (181)$$

2.6 Application of Kifilideen Trinomial Theorem using Matrix Approach

[i] Evaluate the amount of money accumulated after 3 years when \$1 is deposited in a bank paying an annual interest rate of 26 %, compounded yearly using kif trinomial theorem. of a compound 1.26³ using kifilideen trinomial theorem

Solution

$$\text{Amount of money accumulated} = P \left(1 + \frac{R}{100}\right)^T \quad (182)$$

From the question, P = \$1, R = 26 % and T = 3 years

$$\text{Amount of money accumulated} = 1 \left(1 + \frac{26}{100}\right)^3 \quad (183)$$

$$\text{Amount of money accumulated} = 1.26^3 \quad (184)$$

$$\text{Amount of money accumulated} = [1 + 0.2 + 0.06]^3 \quad (185)$$

The combination power for the expansion of the trinomial expression of power 3 in ascending order of terms are 300, 210, 120, 030, 201, 111, 021, 102, 012 and 003

$$1.26^3 = [1 + 0.2 + 0.06]^3 = {}_{3,0,0}^3C 1^3 0.2^0 0.06^0 + {}_{2,1,0}^3C 1^2 0.2^1 0.06^0 + {}_{1,2,0}^3C 1^1 0.2^2 0.06^0 + {}_{0,3,0}^3C 1^0 0.2^3 0.06^0 + {}_{2,0,1}^3C 1^2 0.2^0 0.06^1 + {}_{1,1,1}^3C 1^1 0.2^1 0.06^1 + {}_{0,2,1}^3C 1^0 0.2^2 0.06^1 + {}_{1,0,2}^3C 1^1 0.2^0 0.06^2 + {}_{0,1,2}^3C 1^0 0.2^1 0.06^2 + {}_{0,0,3}^3C 1^0 0.2^0 0.06^3 \quad (186)$$

$$1.26^3 = 1 + 0.6 + 0.12 + 0.008 + 0.18 + 0.072 + 0.0072 + 0.0108 + 0.00216 + 0.000216 \quad (187)$$

$$1.26^3 = 2.000376 \quad (188)$$

3. Discussion

The Kifilideen matrix approach in solving the trinomial expression of power n does not involve the repeated multiplication of figures unlike the long multiplication approach. This eliminates the boring nature of using binomial theorem to expand trinomial of power n .

4. Conclusion

This study implemented Kifilideen trinomial theorem based on matrix approach in computation of the power of base of tri-digits number. This method adopts and employs Kif matrix approach in generating the power combination required in the evaluation. The use of Kifilideen trinomial theorem in expanding trinomial expression of positive power of n help to eliminate humdrum nature of using binomial theorem in expanding such trinomial expression of positive power of n . The Kifilideen matrix approach in solving the tri – digits number of power n does not involve the repeated multiplication of figures unlike the use of long multiplication approach in solving such problem.

Conflict of Interest

The author declares that there is no conflict of interest.

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