

**Article Info**Received: 24th October 2020Revised: 2nd February 2021Accepted: 4th February 2021

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shamssdeeny@gmail.comCite this: *CaJoST*, 2021, 1, 83-90

A Product-Log-Type Estimator of Population Mean in the Presence of Non-Response

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In this paper, a class of log-type estimator under simple random sampling has been suggested, to estimate the finite population mean by using single auxiliary variable under non-response. The proposed estimator, under non-response in different cases; on both the study and auxiliary variables has been studied and gives minimum mean square error as compared to all other considered estimators. Some other estimators are also identified from the proposed class of estimators. We used two real data sets to obtain the efficiencies of the estimators.

Keywords: Auxiliary variables, log type, Mean Square Error (MSE), efficiency, non-response.

1. Introduction

In survey sampling, it is assumed that all the observations are correctly measured on the characteristics under study. But in practice, when we fail to collect the complete information on different variables, non-response is supposed to occur. Non-response occurs due to many reasons, which includes the lack of information provided by respondents, also some of the respondents refuse to answer the questionnaire, sometimes it is difficult to find out the respondents, etc. The gravity of the problem arising from non-response is that it introduces selection bias in addition to causing an increase in variance. For example, if high income earners are less inclined to answer, they are likely to be under represented in the final data set. This implies that the data set tends to contain too few high values and too many low values on any study variable positively correlated with income, such as saving or consumer durable expenditures. The standard estimators of a population total will then yield underestimates. Similarly in agricultural surveys if a particular questionnaire is mailed to all farmers in a region, the non-response rate may be higher among the farmers holding small areas of land. While non-response cannot be completely eliminated in practice, it could be overcome to a great extent by persuasion through repeated visits or making more efforts to collect information by taking a sub-sample of units not responding in the first attempt. Generally auxiliary information is used to increase the precision of the estimators, when there exists a correlation between the study and the auxiliary variables. Ratio, product and regression estimators are good examples in this

context. In daily life there are many situations when we are unable to access the complete information either on the study variable or the auxiliary variable or at the same time both on the study and the auxiliary variables. [2] were the first to suggest a non-response handling technique in mail surveys combined the advantages of mailed questionnaires and personal interviews. Later on, several authors selected the subsample of non-respondents, where the sub-sampling fraction varied according to the non-response rates. [12] Suggested a ratio type estimator for population mean, in the presence of non-response for two-phase sampling, when the population mean of the auxiliary variable x was unknown and non-response occurred on the auxiliary variable. [4], [5]-[7] and [13] suggested different ratio and product types estimators for estimation of population mean using the auxiliary information under non-response. Similarly, [14] and [15] have made significant contributions and proposed ratio, product and difference classes of estimators under non-response. Now, we explain the [2] strategy for non-response;

Suppose a finite population $U = (1, 2, \dots, N)$ of size N units can be divided into two classes $N = N_1 + N_2$. Let N_1 and N_2 be the number of units in the population that form the response and non-response classes respectively. We draw a sample of size n units from U by using a simple random sample without replacement (SRSWOR) sampling scheme. Let r_1 units respond and $r_2 = (n - r_1)$ units do not respond in the first attempt. Let a subsample of size k_2 units be selected

from r_2 non-respondents units, such that $r_2 = k_2 h$; ($h > 1$). Let y_i and x_i be the values of the study variable and auxiliary variable respectively.

Let $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$ and $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ being the sample mean of study variable and auxiliary variable respectively.

2. Notations and Symbols with Selected Estimators

To obtain the properties of estimators, we define the following symbols and notations:

$$E(e_0^*) = E(e_1^*) = E(e_0) = E(e_1) = 0$$

$$E(e_0^{*2}) = \gamma C_Y^2 + \lambda C_{Y(2)}^2 = V_Y^*$$

$$E(e_1^{*2}) = \gamma C_X^2 + \lambda C_{X(2)}^2 = V_X^*$$

$$E(e_1^2) = \gamma C_X^2 = V_X$$

$$E(e_1^* e_1) = E(e_1^2) = \gamma C_X^2 = V_X$$

$$E(e_0^* e_1^*) = \gamma \rho_{YX} + \lambda \rho_{YX(2)} = V_{YX}^*$$

$$E(e_0^* e_1) = E(e_0 e_1) = \gamma \rho_{YX} = V_{YX}$$

where;

$$C_Y^2 = \frac{S_Y^2}{\bar{Y}^2}, C_X^2 = \frac{S_X^2}{\bar{X}^2}, \rho_{YX} = \rho C_X C_Y$$

$$C_{Y(2)}^2 = \frac{S_{Y(2)}^2}{\bar{Y}^2}, C_{X(2)}^2 = \frac{S_{X(2)}^2}{\bar{X}^2}, \rho_{YX(2)} = \rho_{(2)} C_{X(2)} C_{Y(2)}$$

$$\gamma = \left(\frac{1}{n} - \frac{1}{N} \right), \lambda = W_2 \left(\frac{h-1}{n} \right), \text{ and } W_2 = \frac{N_2}{N}$$

3. Review of Some Existing Estimators.

[2] were the first who formulated an unbiased estimator of the population mean \bar{Y} of the study variable Y in the presence of non-response. Initially, they considered the mailed survey in the first attempt and personal interviews in the second attempt after the deadline was over. The

estimator is given by; (1)
where;

$$\bar{y}_{r_1} = \frac{1}{r} \sum_{i=1}^{r_1} y_i \text{ and } \bar{y}_{k_2} = \frac{1}{k_2} \sum_{i=1}^{k_2} y_i$$

The variance of \bar{y}^* , is given by;

$$Var(\bar{y}^*) = \bar{Y}^2 V_y^* \quad (2)$$

The ratio and product estimators under non-response case. When non-response exists on the study variable y as well as on the auxiliary variable x, the traditional ratio and product

estimators for population mean \bar{y}^* and \bar{x}^* are given by;

$$\bar{y}_R^* = \bar{y}^* \left(\frac{\bar{X}}{\bar{x}} \right) \quad (3)$$

$$\bar{y}_P^* = \bar{y}^* \left(\frac{\bar{x}}{\bar{X}} \right) \quad (4)$$

where \bar{y}^* and \bar{x}^* are the [2] estimators for population means \bar{Y} and \bar{X} respectively and are defined by

$$\bar{y}^* = \left(\frac{r_1}{n} \right) \bar{y}_{r_1} + \left(\frac{r_2}{n} \right) \bar{y}_{k_2}, \text{ and } \bar{x}^* = \left(\frac{r_1}{n} \right) \bar{x}_{r_1} + \left(\frac{r_2}{n} \right) \bar{x}_{k_2}$$

with $\bar{y} = (\bar{y}_{r_1}, \bar{x}_{r_1})$ and $(\bar{y}_{k_2}, \bar{x}_{k_2})$ are the sample means of (y, x) based on the samples of r_1 and k_2 units respectively.

The MSEs of \bar{y}_R^* and \bar{y}_P^* to the first order of approximation, are given by;

$$MSE(\bar{y}_R^*) = \bar{Y}^2 (V_Y^* + V_X^* - 2V_{YX}^*) \quad (5)$$

and

$$MSE(\bar{y}_P^{-*}) = \bar{Y}^2 (V_Y^* + V_X^* + 2V_{YX}^*) \tag{6}$$

They have observed that \bar{y}_R^{-*} and \bar{y}_P^{-*} perform better than \bar{y}^{-*} , if $V_{YX}^* > \frac{1}{2}V_X^*$ and $V_{YX}^* < -\frac{1}{2}V_X^*$ respectively.

[12] suggested ratio and product estimators under no-response. When non-response exists only on the study variable y, while the complete information on the auxiliary variable x is available, the ratio and product estimators are given by;

$$\bar{y}_{Rao(R)}^{-*} = \bar{y}^{-*} \left(\frac{\bar{X}}{\bar{x}} \right) \tag{7}$$

and,

$$\bar{y}_{Rao(P)}^{-*} = \bar{y}^{-*} \left(\frac{\bar{x}}{\bar{X}} \right) \tag{8}$$

where \bar{x} is the sample mean \bar{X} based on complete information, and \bar{X} is the population mean of the auxiliary variable.

The MSEs of $\bar{y}_{Rao(R)}^{-*}$ and $\bar{y}_{Rao(P)}^{-*}$ to the first order of approximation, are given by;

$$MSE(\bar{y}_{Rao(R)}^{-*}) = \bar{Y}^2 (V_Y^* + V_X^* - 2V_{YX}^*) \tag{9}$$

and

$$MSE(\bar{y}_{Rao(P)}^{-*}) = \bar{Y}^2 (V_Y^* + V_X^* + 2V_{YX}^*) \tag{10}$$

It is observed that $\bar{y}_{Rao(R)}^{-*}$ and $\bar{y}_{Rao(P)}^{-*}$ perform better than \bar{y}^{-*} , if $V_{YX}^* > \frac{1}{2}V_X^*$ and $V_{YX}^* < -\frac{1}{2}V_X^*$ respectively.

[14] suggested ratio, product and difference type estimators in the case of non-response. They considered the situation in which the population mean of the auxiliary variable x is known, but some units fail to provide information on the

study variable y and the auxiliary variable x. The estimator is given by:

$$\bar{y}_{SK(R1)}^{-*} = \bar{y}^{-*} \left(\frac{\bar{X}}{\bar{x}} \right) \left(\frac{\bar{x}}{\bar{X}} \right) \tag{11}$$

where \bar{x} and \bar{X} both are unbiased estimators of the population mean \bar{X} of the auxiliary variable x.

The MSE of $\bar{y}_{SK(R1)}^{-*}$ to the first order of approximation is given by;

$$MSE(\bar{y}_{SK(R1)}^{-*}) = \bar{Y}^2 (V_Y^* + V_X^* + 3V_X^* - 2(V_{YX}^* + V_{YX}^*)) \tag{12}$$

Note that: the MSE of $\bar{y}_{SK(R1)}^{-*}$ is smaller than \bar{y}^{-*}

if $(V_{YX}^* + V_{YX}^*) > \frac{1}{2}(V_X^* + V_X^*)$. The product estimator of the above mentioned situation is;

$$\bar{y}_{SK(P)}^{-*} = \bar{y}^{-*} \left(\frac{\bar{x}}{\bar{X}} \right) \left(\frac{\bar{X}}{\bar{x}} \right) \tag{13}$$

The MSE of $\bar{y}_{SK(P)}^{-*}$ to the first order of approximation is given by;

$$MSE(\bar{y}_{SK(P)}^{-*}) = \bar{Y}^2 (V_Y^* + V_X^* + 3V_X^* + 2(V_{YX}^* + V_{YX}^*)) \tag{14}$$

Note that the MSE of $\bar{y}_{SK(P)}^{-*}$ performs better

than \bar{y}^{-*} if $(V_{YX}^* + V_{YX}^*) < -\frac{1}{2}(V_X^* + 3V_X^*)$.

4. Suggested Classes of Estimators

Motivated by the series of works carried out in non-response, we therefore suggest a log-type estimator for population mean in the case of non-response, when it exists on the study variable only, and on both the study and the auxiliary variables. Also, we can generate some estimators by substituting different values of ($\delta = 0$ and 1). The proposed estimator(s) is/are given as;

$$T_{\log_y^*}^* = \bar{y}^* + \delta \log \left(\frac{\bar{x}}{\bar{X}} \right) \tag{15}$$

$$T_{\log_{y^* x^*}}^* = \bar{y}^* + \delta \log \left(\frac{x^*}{\bar{X}} \right) \tag{16}$$

Case 1: Non-response exists on the study variable only;

$$T_{\log_y^*}^* = \bar{y}^* + \delta \log \left(\frac{x}{\bar{X}} \right)$$

The Bias, Mean Square Error and optimum MSE of the Proposed Estimator $T_{\log_y^*}^*$ are respectively given as;

$$Bias \left(T_{\log_y^*}^* \right) = -\bar{Y} \frac{\delta}{2} V_X \tag{17}$$

$$MSE \left(T_{\log_y^*}^* \right) = \bar{Y}^2 V_Y^* + 2\delta \bar{Y} V_{YX} + \delta^2 V_X \tag{18}$$

$$MSE \left(T_{\log_y^*}^* \right)_{\min.} = \bar{Y}^2 \left(V_Y^* - \frac{V_{YX}^2}{V_X} \right) \tag{19}$$

Case 2: Non-response exists on both the study variable and auxiliary variable;

$$T_{\log_{y^* x^*}}^* = \bar{y}^* + \delta \log \left(\frac{x^*}{\bar{X}} \right)$$

Expanding $T_{\log_{y^* x^*}}^*$ in terms of e_0^* , e_1^* and e_1 , we have;

$$T_{\log_{y^* x^*}}^* = \bar{Y}(1 + e_0^*) + \delta \log \left(\frac{\bar{X}(1 + e_1^*)}{\bar{X}} \right)$$

$$T_{\log_{y^* x^*}}^* = \bar{Y}(1 + e_0^*) + \delta \log(1 + e_1^*)$$

$$T_{\log_{y^* x^*}}^* = \bar{Y}(1 + e_0^*) + \delta \left(e_1^* - \frac{e_1^{*2}}{2} + \dots \right)$$

$$T_{\log_{y^* x^*}}^* = \bar{Y} + \bar{Y}e_0^* + \delta e_1^* - \delta \frac{e_1^{*2}}{2}$$

$$\left(T_{\log_{y^* x^*}}^* - \bar{Y} \right) = \left(\bar{Y} + \bar{Y}e_0^* + \delta e_1^* - \delta \frac{e_1^{*2}}{2} \right) - \bar{Y}$$

$$\left(T_{\log_{y^* x^*}}^* - \bar{Y} \right) = \bar{Y}e_0^* + \delta e_1^* - \delta \frac{e_1^{*2}}{2}$$

Then taking expectations up to the first order of approximation, we get the Bias of the estimator

$T_{\log_{y^* x^*}}^*$ as;

$$Bias \left(T_{\log_{y^* x^*}}^* \right) = -\bar{Y} \frac{\delta}{2} V_X^* \tag{20}$$

To obtain the MSE of the proposed estimator, we expand the expression;

$$MSE \left(T_{\log_{y^* x^*}}^* \right) = \left(T_{\log_{y^* x^*}}^* - \bar{Y} \right)^2$$

$$MSE \left(T_{\log_{y^* x^*}}^* \right) = \left(\bar{Y}e_0^* + \delta e_1^* \right)^2$$

$$MSE \left(T_{\log_{y^* x^*}}^* \right) = \bar{Y}^2 e_0^{*2} + \delta \bar{Y} e_1^* e_0^* + \delta \bar{Y} e_1^* e_0^* + \delta^2 e_1^{*2}$$

By applying the definition, we have;

$$MSE \left(T_{\log_{y^* x^*}}^* \right) = \bar{Y}^2 V_Y^* + 2\delta \bar{Y} V_{YX}^* + \delta^2 V_X^* \tag{21}$$

To find the optimum value of δ , we differentiate

the $MSE \left(T_{\log_{y^* x^*}}^* \right)$ with respect to δ ;

$$\frac{\partial MSE \left(T_{\log_{y^* x^*}}^* \right)}{\partial \delta} = 0 \Rightarrow 2\delta V_X^* + 2\bar{Y} V_{YX}^* = 0$$

$$2\delta V_X^* = -2\bar{Y} V_{YX}^*$$

$$\delta = \frac{-2\bar{Y} V_{YX}^*}{2V_X^*}$$

$$\delta_{opt.} = -\left(\frac{\bar{Y}V_{YX}^*}{V_X^*}\right)$$

By substituting $\delta_{opt.}$ in the MSE, we get the minimum/optimum MSE of the proposed estimator as;

$$MSE\left(T_{\log_{y^* x^*}}^*\right)_{min.} = \bar{Y}^2\left(V_Y^* - \frac{V_{YX}^{*2}}{V_X^*}\right) \tag{22}$$

Table 1: Members of estimator

Estimators	values of constant	$T_{\log_{y^* x^*}}^*$
$\bar{y} + \log\left(\frac{x^*}{\bar{X}}$	1	
\bar{y}	0	

5. Efficiency Comparisons

In this section we have made theoretical comparisons between the proposed estimator in case of non-response on the study variable only and in the case where there is non-response on both the study and auxiliary variables. Likewise, the proposed estimator in case of non-response on both the study and auxiliary variable is compared to the existing estimators having non-response on both the study and auxiliary variables respectively, which we have discussed earlier.

From (2) and (19), we have;

$$Var\left[\bar{y}^*\right] - MSE\left(T_{\log_{y^* x^*}}^*\right)_{min.} \geq 0$$

$$\bar{Y}^2 V_y^* - \bar{Y}^2\left(V_Y^* - \frac{V_{YX}^{*2}}{V_X^*}\right) \geq 0$$

$$\bar{Y}^2\left(\frac{V_{YX}^{*2}}{V_X^*}\right) \geq 0$$

From (9) and (19), we have;

$$MSE\left[y_{Rao(R)}^*\right] - MSE\left(T_{\log_{y^* x^*}}^*\right)_{min.} \geq 0$$

$$\bar{Y}^2\left(V_Y^* + V_X^* - 2V_{YX}^*\right) - \bar{Y}^2\left(V_Y^* - \frac{V_{YX}^{*2}}{V_X^*}\right) \geq 0$$

$$\bar{Y}^2\left(V_X^* - 2V_{YX}^* + \frac{V_{YX}^{*2}}{V_X^*}\right) \geq 0$$

From (10) and (19), we have;

$$MSE\left[y_{Rao(P)}^*\right] - MSE\left(T_{\log_{y^* x^*}}^*\right)_{min.} \geq 0$$

$$\bar{Y}^2\left(V_Y^* + V_X^* + 2V_{YX}^*\right) - \bar{Y}^2\left(V_Y^* - \frac{V_{YX}^{*2}}{V_X^*}\right) \geq 0$$

$$\bar{Y}^2\left(V_X^* + 2V_{YX}^* + \frac{V_{YX}^{*2}}{V_X^*}\right) \geq 0$$

From (5) and (22), we have;

$$MSE\left[y_R^*\right] - MSE\left(T_{\log_{y^* x^*}}^*\right)_{min.} \geq 0$$

$$\bar{Y}^2\left(V_Y^* + V_X^* - 2V_{YX}^*\right) - \bar{Y}^2\left(V_Y^* - \frac{V_{YX}^{*2}}{V_X^*}\right) \geq 0$$

$$\bar{Y}^2\left(V_X^* - 2V_{YX}^* + \frac{V_{YX}^{*2}}{V_X^*}\right) \geq 0$$

From (6) and (22), we have;

$$MSE\left[y_P^*\right] - MSE\left(T_{\log_{y^* x^*}}^*\right)_{min.} \geq 0$$

$$\bar{Y}^2 (V_Y^* + V_X^* + 2V_{YX}^*) - \bar{Y}^2 \left(V_Y^* - \frac{V_{YX}^{*2}}{V_X^*} \right) \geq 0$$

$$\bar{Y}^2 (V_Y^* + V_X^* + 3V_X^* + 2(V_{YX}^* + V_{YX}^*)) - \bar{Y}^2 \left(V_Y^* - \frac{V_{YX}^{*2}}{V_X^*} \right) \geq 0$$

$$\bar{Y}^2 \left(V_X^* + 2V_{YX}^* + \frac{V_{YX}^{*2}}{V_X^*} \right) \geq 0$$

$$\bar{Y}^2 \left(V_X^* + 3V_X^* + 2(V_{YX}^* + V_{YX}^*) + \frac{V_{YX}^{*2}}{V_X^*} \right) \geq 0$$

From (12) and (22), we have;

$$MSE \left[y_{SK(R1)}^{-*} \right] - MSE \left(T_{\log_{y^* x^*}}^{*} \right)_{\min.} \geq 0$$

6. Empirical Study

To evaluate the performance of the proposed estimator in both cases, we have considered two population data sets as follows;

Population I: Khare and Sinha [10]

$N = 109, n = 30, \bar{Y} = 145.30, \bar{X} = 165.26, \rho = 0.81, C_y = 0.76, C_x = 0.68, w_2 = 0.25, \rho_{(2)} = 0.78, C_{y(2)} = 0.65, C_{x(2)} = 0.57.$

$$\bar{Y}^2 (V_Y^* + V_X^* + 3V_X^* - 2(V_{YX}^* + V_{YX}^*)) - \bar{Y}^2 \left(V_Y^* - \frac{V_{YX}^{*2}}{V_X^*} \right) \geq 0$$

Population II: Khare and Sinha [9]

$N = 96, n = 30, \bar{Y} = 137.92, \bar{X} = 144.87, \rho = 0.77, C_y = 1.32, C_x = 0.81, w_2 = 0.25, \rho_{(2)} = 0.72, C_{y(2)} = 2.08, C_{x(2)} = 0.94.$

$$\bar{Y}^2 \left(V_X^* + 3V_X^* - 2(V_{YX}^* + V_{YX}^*) + \frac{V_{YX}^{*2}}{V_X^*} \right) \geq 0$$

From (14) and (22), we have;

$$MSE \left[y_{SK(P)}^{-*} \right] - MSE \left(T_{\log_{y^* x^*}}^{*} \right)_{\min.} \geq 0$$

We have computed the mean squared errors (MSEs) of the proposed estimators and different existing estimators considered in this study, and the results are displayed in Tables 2 and 3.

Table 2: MSEs of existing and proposed estimators for Population I

Existing Estimators	h=2	h=3	h=4	
y^{-*}	340.4925	412.7758	486.9613	
$y_{Rao(R)}^{-*}$	169.2951	241.5785	351.7640	
$y_{Rao(P)}^{-*}$	937.7810	1010.0643	1084.2498	
y_R^{-*}	125.5447	154.0776	182.6105	
y_P^{-*}	1084.2498	1303.0020	1525.5585	
$y_{SK(R1)}^{-*}$	380.4385	408.9714	437.5043	
$y_{SK(P)}^{-*}$	2107.6294	2326.3816	2548.9381	
t_{BT1y}^{*}	$\Phi = 1$	169.2951	241.5785	351.7640
	$\Phi = 2$	937.7810	1010.0643	1084.2498
	$\Phi = 3$	234.7259	314.5701	396.9073
	$\Phi = 4$	340.4925	412.7758	486.9613
$t_{BT1y^* x^*}^{*}$	$\Phi = 1$	125.5447	154.0776	182.6105
	$\Phi = 2$	1084.2498	1303.0020	1525.5585
	$\Phi = 3$	178.8061	218.7522	258.6982
	$\Phi = 4$	340.4925	412.7758	486.9613

t_{BT1y}^{*-} Min.	171.1973	243.4807	317.6662
$t_{BT1y\ x}^{*-}$ Min.	66.7957	183.5960	191.2837

Table 3: MSEs of existing and proposed estimators for Population II

Existing Estimators	h=2	h=3	h=4	
y^{*-}	1608.7413	2368.7765	3124.5893	
$y_{Rao(R)}^{*-}$	1127.3856	1887.4208	2645.3449	
$y_{Rao(P)}^{*-}$	2723.4596	3483.4949	4241.4189	
y_R^{*-}	785.3697	1207.6115	1625.6309	
y_P^{*-}	3377.9344	4788.2220	6194.2872	
$y_{SK(R1)}^{*-}$	937.3768	1359.6186	1777.6380	
$y_{SK(P)}^{*-}$	5126.0155	6536.3031	7942.3683	
t_{BT1y}^{*-}	$\Phi = 1$	1127.3856	1887.4208	2645.3449
	$\Phi = 2$	2723.4596	3483.4949	4241.4189
	$\Phi = 3$	1353.2850	2113.3202	2871.2442
	$\Phi = 4$	1608.7413	2368.7765	3124.5893
$t_{BT1y\ x}^{*-}$	$\Phi = 1$	785.3697	1207.6115	1625.6309
	$\Phi = 2$	3377.9344	4788.2220	6194.2872
	$\Phi = 3$	1188.6107	1786.0828	2379.3325
	$\Phi = 4$	1608.7413	2368.7765	3124.5893
t_{BT1y}^{*-} Min.	1045.7522	1805.0837	2563.0077	
$t_{BT1y\ x}^{*-}$ Min.	760.4876	1093.5496	1476.4293	

7. Conclusion

We proposed a generalized class of product log-type estimator for estimating the population mean using information on single auxiliary variable under non-response in simple random sampling. Expressions for bias and MSE of the proposed generalized class of estimators are derived up to the first degree of approximation. The proposed estimator is compared with [2] estimator and other considered estimators. A numerical study is carried out to support the theoretical results. In Tables 2 and 3, the

$$MSE\left(T_{\log_{y\ x}^{*-}}^{*}\right)_{\min.}$$

performance of the estimator is efficiently same as the usual linear regression estimator in this aspect, which is the most efficient of all the considered related estimators in literature.

Conflict of interest

The authors declare no conflict of interest.

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