Caliphate Journal of Science & Technology (CaJoST)

ISSN: 2705-313X (PRINT); 2705-3121 (ONLINE)

Research Article

Article Info

Received: 15th December 2020

Revised: 5th January 2021

Accepted: 5th January 2021

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Cite this: *CaJoST*, 2021, 1, 62-68

Daubechies Wavelet-based Galerkin Method of Solving Partial Differential Equations

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In this work, we considered wavelet analysis and the application of Daubechies wavelet-based Galerkin method (DWGM) in solving partial Differential equations (PDEs). It is based on the Galerkin method as it replaces the shape function in the residual equation with the scaling function of the Daubechies wavelet, then the inner product operation is performed in order to obtain a system of equations, which is obtained after considering some properties of the scaling function. The method was tested on some selected problems involving one dimensional equation with Dirichlet boundary conditions. The resulting numerical evidence showed that DWGM is an efficient and effective solver of PDEs as the results exhibited a favourable and faster convergence to the exact solution. Also, results were compared to those of Finite Difference Method (FDM) as seen in literatures.

Keywords: Wavelets, Daubechies Wavelet-based Galerkin Method, Scaling functions, Connection coefficients, Orthogonality

1. Introduction

Many situations and problems of the world today are modelled with partial differential equations (PDEs), therefore, differential equations are very significant in our existence [2]. Many partial differential equations are difficult to resolve analytically, hence, many scholars have developed different numerical methods for solving differential equations [3]. Some recent applications of these numerical methods are New Homotopy Perturbation Method [4], Homotopy Perturbation Method [5], Finite Difference Method [6,7], In [8] Mamadu-Njoseh polynomials was considered in seeking the numerical solutions of fifth-order boundary value Problems, and lots more.

The Daubechies Wavelet-based Galerkin method (DWGM) is among the most recent numerical methods for solving all kinds of partial differential equations, it was developed by Ingrid Daubechies in 1988, and it is based on the Galerkin method [9].The interesting features of wavelet have made it clear to many researchers that it produces efficient and effective numerical approximations to partial differential equations when compared to analytical method and some other numerical methods, this is due to the fact that solutions of many partial differential equations have some local properties like formation of shock and hurricanes, and interactions between scales (an example of this is the atmospheric turbulence) which are also properties of wavelets [10]. The DWGM generates an approximate solution to differential equations by replacing the shape function of the Galerkin method with the scaling function of wavelets [1]. This method has been tested and confirmed by many researchers, due to some features possessed by its wavelets, these features include locality in space (compact support), and locality in scale (vanishing moment) [11,12].

To the best of the knowledge of the authors, of all the works in literatures, there has been no work where the results of the DWGM were compared to the Finite Difference Method (FDM), the closest was [12], but their consideration was on the Modified Daubechies wavelet-based Galerkin method. Some of the applicable properties of the Daubechies Wavelets as recorded in [13] include:

(i.) The area of the scaling function is 1.

This implies that

$$\int_{-\infty}^{\infty} \tau(x)\, dx = 1 \qquad (1)$$

This is so that the scaling function must be normalized. This condition therefore provides for the normalization condition

$\sum_{r=0}^{N-1} a_r = 2$ (2)

Hence, the scaling function and the mother wavelet function of this form are

called a multiplier 2 system, bringing about the $\tau^{\frac{j}{2}}$ term in the Daubechies wavelet transform (DWT) as a normalizer.

(ii.) Both the scaling function and its translates are orthogonal.

This implies that

$$\int_{-\infty}^{\infty} \tau(x-r)\tau(x-p)dx = \delta_{p,r},\ p, r \in Z \quad (3)$$

This condition ensures that

$$\sum_{r=0}^{N-1} a_r a_r - 2m = \delta_{0m},\ m = 0,1,\ldots,\frac{N}{2}-1 \quad (4)$$

This is the orthonormal condition, for coefficients which satisfy (3) and (4),

the functions which are made up of the translates and dilations of the wavelet

function $\varepsilon(2^j x - r)$ form the complete orthogonal basis for $L^2(R)$.

(iii.) Both the scaling function and the wavelet functions are orthogonal.

This implies that

$$\int_{-\infty}^{\infty} \tau(x)\varepsilon(x-r)dx = 0,\ r \in Z \quad (5)$$

[iv] The wavelet function has $\frac{L}{2}$ vanishing moments.

This implies that

$$\int_{-\infty}^{\infty} x^m \varepsilon(x)dx = 0,\ m = 0,1,\ldots,\frac{L}{2} \quad (6)$$

These four conditions buttress that $V_{j+1} = V_j \oplus W_j$ on each fixed scale j .

The wavelets $\{\varepsilon_{jr}(x)\}$, $r \in Z$ form an orthonormal basis W_j, and the scaling

functions $\{\varepsilon_{jr}(x)\}$, $r \in Z$ form an orthonormal basis V_j. V_j is the multiresolution analysis of $L^2(R)$.

This paper reiterates the significance and application of the DWGM in solving PDEs, and compares the obtained results with that of the exact solution and the Finite Difference Method (FDM) in literatures.

2. Daubechies Wavelet-based Galerkin Method

Some of the important components of the DWGM and their roles are considered below.

2.1 Scaling functions and Mother wavelet functions

In [4] the scaling function was defined as

$$\tau(x) = \sum_{r=0}^{N-1} a_r \tau(2x-r) \quad (7)$$

where $a_r, (r = 0,1,\ldots,N-1)$ are the filter coefficients, N (which is an even integer) is the genus of the wavelet. The functions which are generated with the filter coefficients have their supp(τ) as $[0, N-1]$ and their vanishing moments as $[\frac{N}{2}-1]$ [1].

In [13] also the mother wavelet function was defined as

$$\varepsilon(x) = \sum_{r=N}^{1} (-1)^r a_{(1-r)} \tau(2x-r) \quad (8)$$

and it was stated in [1] that every filter coefficient of the scaling functions and the mother wavelets must satisfy the two conditions below

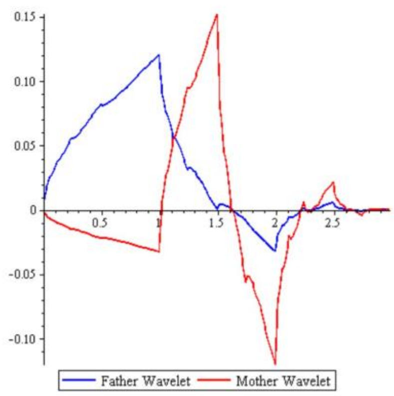
(i.) $\sum_{r=0}^{N-1} a_r = 2$
(ii.) $\sum_{r=0}^{N-1} a_r a_{r-s} = 2\delta_{0,s}$

Where $\delta_{0,s}$ is the Kronecker Delta.

A graphical description of the scaling function and the mother wavelet function is given in Figure 1.

Figure 1: The scaling function and the mother wavelet function

2.1.1 Computing the Scaling function



According to [9], the scaling function computation is almost the same as the problem of Eigen vector and Eigen value. For instance, computing the scaling function (equation (7)) for Daubechies-8 (that is when N = 8) yields

$\tau(0.0) = a_0\tau(0) + a_1\tau(-1) + a_2\tau(-2) + a_3\tau(-3) + a_4\tau(-4) + a_5\tau(-5) + a_6\tau(-6) + a_7\tau(-7)$

$\tau(0.5) = a_0\tau(1) + a_1\tau(0) + a_2\tau(-1) + a_3\tau(-2) + a_4\tau(-3) + a_5\tau(-4) + a_6\tau(-5) + a_7\tau(-6)$

$\tau(1) = a_0\tau(2) + a_1\tau(1) + a_2\tau(0) + a_3\tau(-1) + a_4\tau(-2) + a_5\tau(-3) + a_6\tau(-4) + a_7\tau(-5)$

$\tau(1.5) = a_0\tau(3) + a_1\tau(2) + a_2\tau(1) + a_3\tau(0) + a_4\tau(-1) + a_5\tau(-2) + a_6\tau(-3) + a_7\tau(-4)$

$\tau(2) = a_0\tau(4) + a_1\tau(3) + a_2\tau(2) + a_3\tau(1) + a_4\tau(0) + a_5\tau(-1) + a_6\tau(-2) + a_7\tau(-3)$

$\tau(2.5) = a_0\tau(5) + a_1\tau(4) + a_2\tau(3) + a_3\tau(2) + a_4\tau(1) + a_5\tau(0) + a_6\tau(-1) + a_7\tau(-2)$

$\tau(3.0) = a_0\tau(6) + a_1\tau(5) + a_2\tau(4) + a_3\tau(3) + a_4\tau(2) + a_5\tau(1) + a_6\tau(0) + a_7\tau(-1)$

$\tau(3.5) = a_0\tau(7) + a_1\tau(6) + a_2\tau(5) + a_3\tau(4) + a_4\tau(3) + a_5\tau(2) + a_6\tau(1) + a_7\tau(0)$

$\tau(4.0) = a_0\tau(8) + a_1\tau(7) + a_2\tau(6) + a_3\tau(5) + a_4\tau(4) + a_5\tau(3) + a_6\tau(2) + a_7\tau(1)$

$\tau(4.5) = a_0\tau(9) + a_1\tau(8) + a_2\tau(7) + a_3\tau(6) + a_4\tau(5) + a_5\tau(4) + a_6\tau(3) + a_7\tau(2)$

$\tau(5.0) = a_0\tau(10) + a_1\tau(9) + a_2\tau(8) + a_3\tau(7) + a_4\tau(6) + a_5\tau(5) + a_6\tau(4) + a_7\tau(3)$

$\tau(5.5) = a_0\tau(11) + a_1\tau(10) + a_2\tau(9) + a_3\tau(8) + a_4\tau(7) + a_5\tau(6) + a_6\tau(5) + a_7\tau(4)$

$\tau(6.0) = a_0\tau(12) + a_1\tau(11) + a_2\tau(10) + a_3\tau(9) + a_4\tau(8) + a_5\tau(7) + a_6\tau(6) + a_7\tau(5)$

$\tau(6.5) = a_0\tau(13) + a_1\tau(12) + a_2\tau(11) + a_3\tau(10) + a_4\tau(9) + a_5\tau(8) + a_6\tau(7) + a_7\tau(6)$

$\tau(7.0) = a_0\tau(14) + a_1\tau(13) + a_2\tau(12) + a_3\tau(11) + a_4\tau(10) + a_5\tau(9) + a_6\tau(8) + a_7\tau(7)$

$\tau(7.5) = a_0\tau(15) + a_1\tau(14) + a_2\tau(13) + a_3\tau(12) + a_4\tau(11) + a_5\tau(10) + a_6\tau(9) + a_7\tau(8)$

Recall that the functions generated with filter coefficients have supp[0, N −1] = [0, 8 − 1] = [0, 7],

therefore,

$\tau(-1) = \tau(-2) = \tau(-3) = \tau(-4) = \tau(-5) = \tau(-6) = \tau(-7) = \tau(8) = \tau(9) = \tau(10) = \tau(11) = \tau(12) = \tau(13) = \tau(14) = \tau(15) = 0$

.

Recall also that $a_0 \neq 0$ for $r = 0,1,\dots,7$.

This implies that $\tau(0) = \tau(7) = 0$

In general, the result becomes,

$\tau(0) = 0$

$\tau(0.5) = a_0\tau(1)$

$\tau(1.0) = a_0\tau(2) + a_1\tau(1)$

$\tau(1.5) = a_0\tau(3) + a_1\tau(2) + a_2\tau(1)$

$\tau(2.0) = a_0\tau(4) + a_1\tau(3) + a_2\tau(2) + a_3\tau(1)$

$\tau(2.5) = a_0\tau(5) + a_1\tau(4) + a_2\tau(3) + a_3\tau(2) + a_4\tau(1)$

$\tau(3.0) = a_0\tau(6) + a_1\tau(5) + a_2\tau(4) + a_3\tau(3) + a_4\tau(2) + a_5\tau(1)$

$\tau(3.5) = a_1\tau(6) + a_2\tau(5) + a_3\tau(4) + a_4\tau(3) + a_5\tau(2) + +a_6\tau(1)$

$\tau(4.0) = a_2\tau(6) + a_3\tau(5) + a_4\tau(4) + a_5\tau(3) + a_6\tau(2) + a_7\tau(1)$

$\tau(4.5) = a_3\tau(6) + a_4\tau(5) + a_5\tau(4) + a_6\tau(3) + a_7\tau(2)$

$\tau(5.0) = a_4\tau(6) + a_5\tau(5) + a_6\tau(4) + a_7\tau(3)$

$\tau(5.5) = a_5\tau(6) + a_6\tau(5) + a_7\tau(4)$

$\tau(6.0) = a_6\tau(6) + a_7\tau(5)$

$\tau(6.5) = a_7\tau(6)$

$\tau(7.0) = 0$

We write the non-trivial equations in matrix form as $\bar{\tau} = A\bar{\tau}$.

2.2 Multiresolution in $L^2(R)$

In [13] multiresolution analysis was defined as a sequence of closed subspaces $\{V_j : j \in Z\}$, which satisfies the stated conditions below. Kalia [9] also buttressed that a formal approach of construction of orthogonal bases of wavelets is the multiresolution analysis, according to him, it uses some rules and steps, and it is made up of a sequence $\{V_j : j \in Z\}$ of embedded closed subspace of the $L^2(R)$ space and satisfies the following conditions

(1.) $V_j \subset V_{j+1} \subset L^2(R) \forall j \in Z$
(2.) $\cup_{j=-\infty}^{\infty} V_j = L^2(R)$
(3.) $\cap_{j=-\infty}^{\infty} V_j = 0$
(4.) $f(x) = V_j \equiv f(x) = V_{j+1}$
(5.) $\exists \tau(x) \in V_0 \ni \{\tau_{0,n} = \tau(x-n), n \in Z\}$ is an orthogonal basis for V_0.

Here, τ is the scaling function, it will generate the multiresolution analysis if $\{V_j\}$ is a multiresolution of $L^2(R)$ and V_0 is the closed subspace which is generated by the integer translates of a single function.

The translates of the mother wavelet function and the scaling function define orthogonal basis which are generated in each V_j by

$$V_j = Span\{\tau_{j,k}(x)\}, k \in Z$$

$$W_j = Span\{\varepsilon_{j,k}(x)\}, k \in Z$$

where

$$\tau_{j,k}(x) = 2^{\frac{j}{2}}\tau(2^j x - k), j, k \in Z$$ and

$$\varepsilon_{j,k}(x) = 2^{\frac{j}{2}}\tau(2^j x - k), j, k \in Z$$

Therefore, W_j is an orthogonal complement of V_j and a scaling function $\tau(x)$ must exist in order to obtain a basis in each V_j. Since W_j represents a subspace complementing the subspace V_j in V_{j+1}, which means that $V_{j+1} = V_j \oplus W_j$.

We can write each element of V_{j+1} uniquely as the sum of elements in V_j, and the element having the required details of moving from one level of approximation j to another level $j+1$ of approximation is in W_j. For clarity, considering the scaling function $\tau(x)$, the mother wavelet $\varepsilon(x)$ can be constructed in the form

$$W_j = Span\{\varepsilon_{j,k}(x) = 2^{\frac{j}{2}}\tau(2^j x - k), j, k \in Z\} \quad (9)$$

2.3 Connection Coefficients

As explained in [14], when using the wavelet-Galerkin method to find the solutions of partial differential equations, we must compute the connection coefficients.

$$\ell_{t_1 t_2}^{q_1 q_2} = \int_{-\infty}^{\infty} \tau^{q_1}(x - t_1)\tau^{q_2}(x - t_2)dx \quad (10)$$

If we differentiate the scaling function q times, we will obtain

$$\tau^q(x) = 2^q \sum_{r=0}^{N-1} a_r \tau^q(2x - r) \quad (11)$$

After the simplification and considering it for each $\ell_{t_1 t_2}^{q_1 q_2}$, we will obtain a system of linear equations having $\ell^{q_1 q_2}$ as an unknown vector.

$$C\ell^{q_1 q_2} = \frac{1}{2^{q-1}}\ell^{q_1 q_2} \quad (12)$$

where $q = q_1 + q_2$, and $C = \sum_i a_i a_s - 2t + 1$.

We can obtain the moments M_i^j of the translates of $\tau(x)$ using the formula

$$M_i^j = \int_{-\infty}^{\infty} x^j \tau(x - i)dx \quad (13)$$

where $M_0^0 = 1$.

In [14] the formula for computing moments of translates of the scaling function $\tau(x)$ was derived, which they achieved by induction on k.

In computing the connection coefficients and moments at different scales, we obtain the connection coefficients by substituting the Daubechies coefficients in the matrix C and evaluate the moments.

3. Numerical Illustration

3.1 Test Problem 1

Consider the wave equation in [16],

$$u_{xx} + u_x = 0, 0 \le x \le 1 \quad (14)$$

with Dirichlet boundary condition $u(1) = 0$, and $u(0) = 1$, and $\varphi = 0.000002$.

The exact solution of the problem in (14) above

$$u = cos\varphi^{\frac{1}{2}}x - cos\varphi^{\frac{1}{2}}x$$

Using the Daubechies Wavelet-Based Galerkin Method,

Let N = 6, j = 0, we assume u(x) defined as

$$u(x) = \sum_{r=1-N}^{2^j} B_r 2^{\frac{j}{2}}\tau(2^j x - r) \quad (15)$$

to be a solution to (14),

Substituting into the original equation, we have

$$\frac{d^2}{dx^2}\left(\sum_{r=1-N}^{2^j} B_r 2^{\frac{j}{2}}\tau(2^j x - r)\right) + \varphi \sum_{r=1-N}^{2^j} B_r 2^{\frac{j}{2}}\tau(2^j x - r) \equiv \sum_{r=1-N}^{2^j} B_r 2^{\frac{j}{2}}\tau_{xx}(2^j x - r)) + \varphi \sum_{r=1-N}^{2^j} B_r 2^{\frac{j}{2}}\tau(2^j x - r) \neq 0 \quad (16)$$

Let $y = 2^j x$, and $C_r = B_r 2^{\frac{j}{2}}$,

Substituting into equation (16), we have

$$\sum_{r=1-N}^{2^j} C_r \tau_{xx}(y - r) + \varphi \sum_{r=1-N}^{2^j} C_r \tau(y - r) \quad (17)$$

From here, (15) now becomes

$$u(x) = \sum_{r=1-N}^{2^j} C_r \tau(y - r) \quad (18)$$

Next, we perform the inner product operation on (17) with the scaling function, to have

$$\sum_{r=1-N}^{2^j} C_r \int \tau_{xx}(y - r)\tau(y - p)dy + \varphi \sum_{r=1-N}^{2^j} C_r \int \tau(y - r)\tau(y - p)dy \quad (19)$$

$$p = 1 - N, 2 - N, \ldots, 2^j \equiv -5, -4, \ldots, 0, 1.$$

by the orthogonality property of Daubechies wavelet,

$$\int \tau(y - r)\tau(y - p)dy = \delta_{p,r}$$

we have

$$\sum_{r=1-N}^{2^j} C_r \int \tau_{xx}(y - r)\tau(y - p)dy + \varphi \sum_{r=1-N}^{2^j} C_r \quad (20)$$

By the definition of the connection coefficients $\ell_{r,p}$,

$$\ell_{r,p} = \int \tau_{xx}(y - r)\tau(y - p)dy \quad (21)$$

we have

$$\sum_{r=1-N}^{2^j} C_r \ell_{r,p} + \varphi \sum_{r=1-N}^{2^j} C_r \quad (22)$$

Applying the Dirichlet boundary conditions,

$$u(0) = \sum_{r=-5}^{1} C_r \tau(0-r) = 1 \qquad (23)$$

$$u(1) = \sum_{r=-5}^{1} C_r \tau(1-r) = 0 \qquad (24)$$

Equations (23) and (24) (which are the left and right boundary conditions) now become the first and the last lines respectively of (22). Next we obtain the matrix $TC = B$ below with $N = 6$.

$$\begin{pmatrix} 0 & \tau(4) & \tau(3) & \tau(2) & \tau(1) & 0 & 0 \\ \lambda_1 & \lambda_0+\varphi & \lambda_{-1} & \lambda_{-2} & \lambda_{-3} & \lambda_{-4} & \lambda_{-5} \\ \lambda_2 & \lambda_1 & \lambda_0+\varphi & \lambda_{-1} & \lambda_{-2} & \lambda_{-3} & \lambda_{-4} \\ \lambda_3 & \lambda_2 & \lambda_1 & \lambda_0+\varphi & \lambda_{-1} & \lambda_{-2} & \lambda_{-3} \\ \lambda_4 & \lambda_3 & \lambda_2 & \lambda_1 & \lambda_0+\varphi & \lambda_{-1} & \lambda_{-2} \\ \lambda_5 & \lambda_4 & \lambda_3 & \lambda_2 & \lambda_1 & \lambda_0+\varphi & \lambda_{-1} \\ 0 & 0 & \tau(4) & \tau(3) & \tau(2) & \tau(1) & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} C_{-5} \\ C_{-4} \\ C_{-3} \\ C_{-2} \\ C_{-1} \\ C_0 \\ C_1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Solving with Gaussian elimination method, using Maple software, we get

$$C = \begin{pmatrix} C_{-5} \\ C_{-4} \\ C_{-3} \\ C_{-2} \\ C_{-1} \\ C_0 \\ C_1 \end{pmatrix} = \begin{pmatrix} 3.8385 \\ 4.3212 \\ 3.4306 \\ 2.3060 \\ 1.2008 \\ 0.1781 \\ -0.450 \end{pmatrix}$$

To get the solution, we substitute C_r directly into (18), this gives the approximate solution as

$$u = cos\varphi x - cot\varphi^{\frac{1}{2}} x$$

The FDM result of this problem has been generated with Maple 18 software

as in table 1 below.

3.2. Test Problem 2

Consider the equation

$$u_{tt} - u_t = -t, 0 \le t \le 1$$

with boundary conditions $u(0) = 0, u(1) = 1$,

the exact solution of the equation is given as $u(t) = \frac{\sin(t)}{\sin(1)} - t.$

Solving with DWGM as explained in section 3.1, yields

$$u(t) = \left(\frac{1}{\sin(1)} - 1\right) t - \left(\frac{1}{6\sin(1)}\right) t^3 + \left(\frac{1}{120\sin(1)}\right) t^5$$

The FDM result of this problem has been extracted from [15].

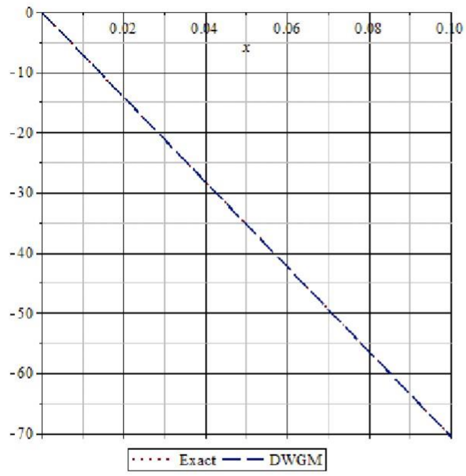
3.3. Tabular and Graphical Comparison of Results

Table 1: Numerical results for Test Problem 1, using $i = \frac{x}{10}$

$i = \frac{x}{10}$	Exact Solution	FDM Solution	Error from FDM	DWGM Solution	Error from DWGM
0.01	-7070.0677670	-7070.0677670	0.0000000	-7070.0677670	0.0000000
0.02	-3534.5338130	-3534.5338130	0.0000000	-3534.5338130	0.0000000
0.03	-2356.0224630	-2356.0224630	0.0000000	-2356.0224630	0.0000000
0.04	-1766.7667650	-1766.7667650	0.0000000	-1766.7667650	0.0000000
0.05	-1413.2133270	-1413.2133270	0.0000000	-1413.2133270	0.0000000
0.06	-1177.5110190	-1177.5110190	0.0000000	-1177.5110190	0.0000000
0.07	-1009.1522150	-1009.1522150	0.0000000	-1009.1522150	0.0000000
0.08	-882.88309990	-882.88309990	0.0000000	-882.88309990	0.0000000
0.09	-784.67309990	-784.67309990	0.0000000	-784.67309990	0.0000000
0.10	-706.10631100	-706.10631100	0.0000000	-706.10631100	0.0000000

Table 2: Numerical results for Test Problem 2, using $i = \frac{t}{10}$

$i = \frac{t}{10}$	Exact Solution	FDM Solution	Error from FDM	DWGM Solution	Error from DWGM
0.01	0.0186415	0.018660	1.80e-05	0.0186415	1.0000e-10
0.02	0.0360977	0.036132	3.40e-05	0.0360977	3.0000e-09
0.03	0.0511948	0.051243	4.80e-05	0.0511948	5.1500e-08
0.04	0.0627829	0.062842	5.90e-05	0.0627832	3.8560e-07
0.05	0.0697470	0.069812	6.50e-05	0.0697488	1.8358e-06
0.06	0.0710184	0.071084	6.60e-05	0.0710249	6.5679e-06
0.07	0.0655851	0.065646	6.10e-05	0.0656044	1.9287e-05
0.08	0.0525025	0.052550	4.80e-05	0.0525515	4.9012e-05
0.09	0.0309019	0.030903	2.80e-05	0.0310134	1.1152e-04



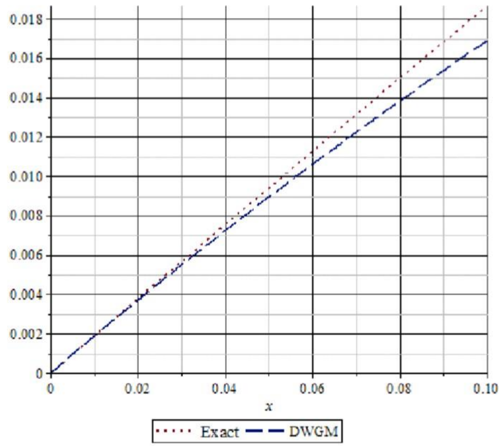


Figure 3: Graphical Result for test problem 2

Figure 2: Graphical presentation of results of test problem 1

4. Discussion of Results

We have successfully implemented the Daubechies Wavelet-based Galerkin method (DWGM) for the solutions of linear parabolic Partial Differential Equations, and the following observations were recorded.

(i.) In test problem 1, it was observed that DWGM attained a maximum error of order 10^{-6}, which is the same maximum error attained by the FDM. Also, the results presented graphically as shown in Figure 2 revealed that the DWGM and the FDM have the same rate of convergence.

(ii.) From the above, we can see that the DWGM converges (as well as the FDM) favourably to the exact solution.

(iii.) Unlike the case of test problem 1, we can observe from the table of results for test problem 2 that the DWGM attained a maximum error of 10^{-10}, while the FDM attained maximum error of 10^{-5}, this clearly shows that the DWGM converges more favourably to the exact solution than the FDM [15]. Also the presented graph as shown in Figure 3 buttressed the more favourable convergence of the DWGM to the exact solution than the FDM in literature.

5. Conclusion

The results of this work have shown that the DWGM is effective in approximating solutions of partial differential equations and it is hugely accurate. The result has also shown some advantages such as

(i.) Excellent convergent rate;
(ii.) Ease of application and implementation; and
(iii.) Effectiveness and efficiency in approximating solutions to Partial Differential Equations.

Acknowledgements

The authors wish to acknowledge the efforts and encouragements of all the lecturers and staff of the Department of Mathematics, Delta State University, Abraka, Nigeria.

Competing interests

The authors declare that they have no competing interests.

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