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Combined Effects of Radiative Heat Source and Magnetic Field on a Free Convection Flow in a Moving Vertical Plate Filled With Porous Material

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Free convection heat transfer flow of a moving permeable vertical plate has been contemplated by incorporating heat source in the presence of thermal radiation, magnetic field and porous material. The non-dimensional governing equations of the velocity, temperature, skin friction as well as Nusselt number are solved analytically subject to a set of appropriate boundary conditions using perturbation method. The solutions obtained are graphically represented and the effect of various controlling parameters such as thermal radiation (R), heat source parameter (Z), magnetic field (m) and Darcy number (Da) are discussed in detail. It is interesting to report that, by varying these pertinent parameters, the fluid flow is substantially influenced.

Keywords: Free convection, Thermal radiation, MHD, Heat source, Porous medium

1. Introduction

There has been a renewed interest and increasing research activities in free convective flow problems due to its diverse practical engineering and industrial applications. Some reported applications are in geophysics, aeronautics, nuclear power reactors and soil sciences. Ahmad *et al.* [1] presented free convection flow with diffusion-thermo and Navier slip condition in the presence of Newtonian heating in a vertical porous channel. The natural convection heat and mass transfer in MHD fluid flow past a moving vertical plate with variable surface temperature and concentration in a porous medium was reported by Javaherdeh *et al.* [2]. Vanita and Kumar [3] analyzed heat and mass transfer flow of radial magnetic field on free convective flow over ramped velocity moving vertical cylinder. Research works in the area of natural convective heat transfer flow through porous medium has substantially been carried out in recent times. This is primarily because of numerous practical applications of these flows in engineering and sciences, such as storage of radioactive nuclear waste materials transfer, separation processes in chemical industries, filtration, transpiration cooling, transport processes in aquifers, ground water pollution, etc [4]. Natural convection saturated with porous medium has been reported in many literatures by some authors through various boundaries like in vertical channels [5, 6, 7, 8], in vertical cylinders [9, 10], in inclined plates [11], in rectangular enclosures [12] just to cite a few among many others.

Free convection-radiation in channels formed by vertical plates has received attention among researchers in the last few decades due to its widespread importance in engineering applications like cooling of electronic equipment, design of passive solar systems for energy conversion, design of heat exchangers, human comfort in buildings, thermal regulation processes and many more. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment [13], therefore its effects has been studied through various channels [14, 15, 16, 17, 18]. The results of some of these findings revealed that both velocity and temperature increase significantly with increase in thermal radiation parameter and time. The effects of thermal radiation and free convection currents on the unsteady Couette flow between two vertical parallel plates with constant heat flux at one boundary have been studied by Narahari [19]. Unsteady free convective Couette flow of heat generating/absorbing fluid in porous medium has been investigated by Deka and Bhattacharya [20]. Kumar and Varma [21] have examined the radiation effects on MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature in the presence of heat generation.

In recent years, researchers have shown interest in this emerging field of magneto-hydrodynamics

(MHD) due to its wide range of applications in astrophysics, geophysics and engineering. These types of studies are important in nuclear power plants, drilling operations, geothermal power generation and space vehicles technology etc [3]. Keeping this in view, some authors have carried out investigations through various layer boundaries [22, 23, 24, 25] and they have reported that magnetic field effect can be used as an effective tool for suppressing fluid flow. The main purpose of this paper is to carry out steady-state analysis of the fully developed free convective heat transfer flow of a moving vertical plate under the influence of thermal radiation and magnetic field effects with heat generation saturated with porous medium. The dimensional governing equations are transformed using the non-dimensional quantities and the resultant dimensionless equations are solved by perturbation technique method.

2. Mathematical Analysis

The effects of radiation and magnetic field of a steady fully developed free convection flow of a viscous incompressible fluid between two infinite vertical parallel plates separated by a distance h , in the presence of heat source and porous material has been analyzed. The x' -axis is taken along the plates in the vertical upwards direction and the y' -axis is taken normal to it. The plate at $y=0$ begins to move as it is heated by the buoyancy force arising from the temperature gradient occurring as a result of asymmetric heating of the plate while the plate at $y=h$ is stationary and is maintained with constant temperature θ_h . The plates are considered of infinite length in x' -direction and the flow is depending only on y' since it is steady fully developed. A uniform magnetic field of strength B_0 is imposed perpendicular to the plates. It is also assumed that the radiative heat flux in the x -direction is negligible as compared to that in the y -direction. Invoking the usual Boussinesq approximation, the relevant mathematical model capturing the present physical situation in dimensionless form are:

$$\frac{d^2 u}{dy^2} - s \frac{du}{dy} + \theta = \left(m^2 + \frac{1}{Da} \right) u \quad (1)$$

$$(1+R) \frac{d^2 \theta}{dy^2} - s Pr \frac{d\theta}{dy} + Z\theta = 0 \quad (2)$$

Subject to the following initial and boundary condition in dimensionless form:

$$\left. \begin{aligned} U = Lp, \quad \frac{d\theta}{dy} = -1 & \quad \text{at} \quad y = 0 \\ U = 0, \quad \theta = 0 & \quad \text{at} \quad y = 1 \end{aligned} \right\} \quad (3)$$

The above equations are rendered in non-dimensional form by defining the following non-dimensional parameters:

$$\begin{aligned} u &= \frac{u'}{u_0}, \quad y = \frac{y'}{H}, \quad \theta = \frac{\theta' - \theta_0}{\theta_w - \theta_0}, \\ Da &= \frac{k_0}{b^2}, \quad m^2 = \frac{\sigma B_0^2 y_0^2}{\rho \nu}, \quad R = \frac{4Ih^2}{k}, \\ Z &= \frac{Ih^2}{k}, \quad Pr = \frac{\rho \nu c_p}{k}, \quad s = \frac{V_0 H}{\nu} \end{aligned} \quad (4)$$

3. Method of Solution

In order to construct approximate solutions to equations (1) and (2) subject to (3), we employ a regular perturbation method by assuming that the radiation parameter is small and taking the power series expansion in the radiation parameter R , the solution for the velocity and temperature is obtained using:

$$U = u_0 + Ru_1 + \dots + \sum_{i=0}^{\infty} R^i u^i \quad (5)$$

$$\theta = \theta_0 + R\theta_1 + \dots + \sum_{i=0}^{\infty} R^i \theta^i \quad (6)$$

substituting equation (5) and (6) into equation (1) and (2) and equating the coefficients of like powers of R , the solution of the governing equations for the velocity and energy and fields subject to boundary conditions (3) are obtained as follows:

$$\begin{aligned} u(y) &= A_5 e^{m^3 y} + A_6 e^{-m^4 y} + D_3 e^{m^1 y} + D_4 e^{-m^2 y} \\ &+ R(A_7 e^{m^3 y} + A_8 e^{-m^4 y} + D_5 e^{m^1 y} + D_6 e^{-m^2 y}) \end{aligned} \quad (7)$$

$$\begin{aligned} \theta(y) &= A_1 e^{m^1 y} + A_2 e^{-m^2 y} \\ &+ R(A_3 e^{m^1 y} + A_4 e^{-m^2 y} + D_1 e^{m^1 y} + D_2 e^{-m^2 y}) \end{aligned} \quad (8)$$

the expression for the skin friction at the

boundary is obtained as follows:

$$\begin{aligned} \tau = \left[\frac{du}{dy} \right] &= m_3 A_5 e^{m^3} - m_4 A_6 e^{-m^4} + m_1 D_3 e^{m^1} - m_2 D_4 e^{-m^2} \\ &+ R(m_3 A_7 e^{m^3} - m_4 A_8 e^{-m^4} + m_1 D_5 e^{m^1} - m_2 D_6 e^{-m^2}) \end{aligned} \quad (9)$$

The expression for the Nusselt number is

obtained as:

$$\begin{aligned} Nu = \left[\frac{d\theta}{dy} \right] &= m_1 A_1 e^{m^1} - m_2 A_2 e^{-m^2} \\ &+ R[m_1 A_3 e^{m^1} - m_2 A_4 e^{-m^2} + m_1 D_1 e^{m^1} - m_2 D_2 e^{-m^2}] \quad (10) \end{aligned}$$

Where $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, D_1, D_2, D_3, D_4, D_5, D_6$ are defined in the appendix.

Table 1. Showing computational results of Nusselt number and skin friction for varying values of R, Z, Pr, m and K for $s=0.2$

R	Z	Pr	M	K	Nu	τ
0.1	0.1	0.71	0.1	0.01	1.0018	0.0730
0.2	0.1	0.71	0.1	0.01	1.0929	0.0803
0.1	0.2	0.71	0.1	0.01	1.0532	0.1341
0.1	0.1	7	0.1	0.01	0.2901	-0.2890
0.1	0.1	0.71	1	0.01	-	0.0733
0.1	0.1	0.71	0.1	0.02	-	0.1157

4. Results and Discussion

In the present work, the analysis of free convection flow with thermal radiation and magnetic field in a moving vertical plate through a porous medium in the presence of heat source has been studied. The analytical solution for the non-dimensional velocity, temperature, skin friction and Nusselt number has been obtained using perturbation technique. The effect of various flow parameters on the flow field has been studied and the results are presented graphically and discussed. The influence of the controlling parameters such as thermal radiation, R; Darcy number, K; heat source parameter, Z; Prandtl number, Pr and magnetic field, m are demonstrated in Figures 1-7 where L_p is the moving plate at 1. Throughout our computations, unless otherwise stated, we have taken the default values of parameters as $R=0.1, K=0.1, Z=0.01, m=0.1$. Figure 1 describes the behavior of R on temperature distribution and it is abundantly clear that the temperature is substantially accelerated with an increase in R. From Figure 2, the influence of Z on temperature at the moving plate is illustrated and it is clear from the figure that temperature of the fluid is enhanced with increase in the values of Z. Higher values of Prandtl number is seen to depress the temperature profile as elucidated in Figure 3. This implies that an increase in Prandtl number leads to a fall in the thermal boundary layer flow. This is because fluids with large Pr have low thermal diffusivity which causes low heat penetrating resulting in reduced thermal boundary layer. Figure 4 shows the effects of radiation on velocity distribution. It is noticed that, the fluid motion accelerates faster with increase in the values of R. In view of this, Jha *et al.* [15] also reported that increasing radiation parameter brings about enhancement on transient and steady state velocity throughout the channel and that this may be attributed to the fact that as radiation parameter increases, temperature of the fluid

increases which causes higher convection current. Figure 5 demonstrates the effects of heat source parameter on velocity and it is evident from the figure that, the fluid motion is enhanced with increasing values of Z. Figure 6 reveals the effects of Da on the fluid flow at the moving plate. As expected, the fluid motion is seen to increase significantly as the values of Da are increasing. It is revealed from Figure 8 that, increasing magnetic parameter m, vastly suppresses the fluid flow. Physically, $m = 0$ means that there is no magnetic effect and the flow is purely hydrodynamic. It is found from this figure that the velocity decreases with increasing value of m. This is expected as the application of the transverse magnetic field always results in a resistance-type force called Lorentz force. This force is similar in nature to a drag force, and upon increasing the values of m, the drag force increases and tends to resist the fluid flow, thus reducing the fluid motion significantly.

Table 1 displays the numerical results of radiation, heat source parameter, Darcy number, prandtl number and magnetic field effect on skin friction and Nusselt number. From the table, it is clear that, radiation and heat source increase the rate of heat transfer and consequently the sheer stress while retardation is observed for increasing values of Prandtl number. As expected, Darcy number and magnetic effects improves the skin friction at the moving plate respectively.

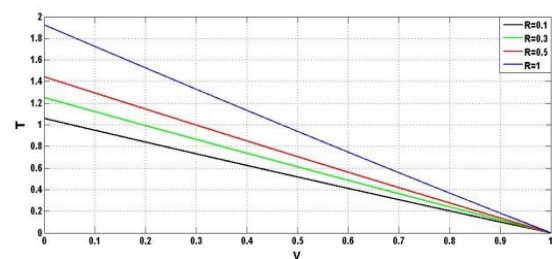


Figure 1. The Effects of Radiation on temperature profile.

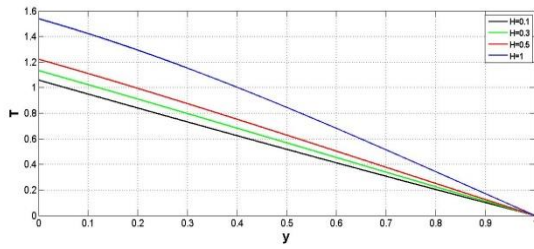


Figure 2. The Effect of Heat source parameter on temperature.

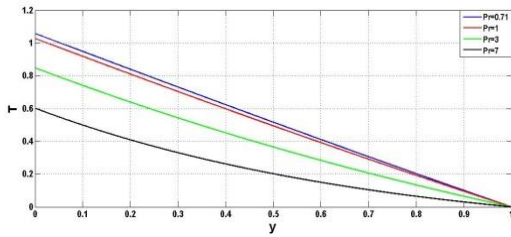


Figure 3. The effect of Prandtl number on temperature.

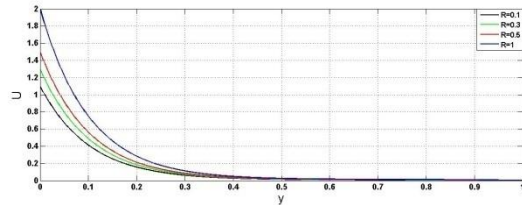


Figure 4. The Effect of Radiation on velocity profile.

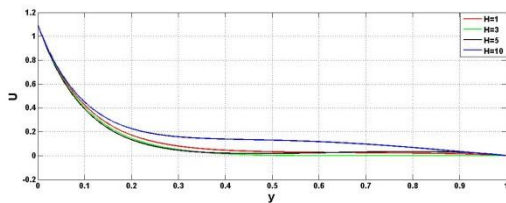


Figure 5. The Effect of Heat source parameter on velocity profile.

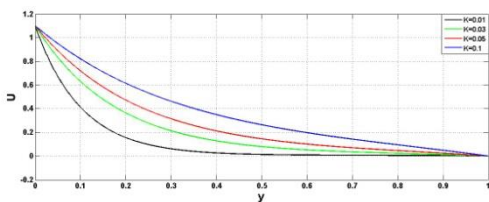


Figure 6. The Effect of Darcy number on velocity profile.

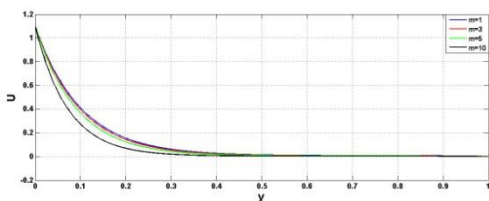


Figure 7. The Effect of magnetic parameter on velocity profile.

5. Conclusion

The MHD free convection flow of a fluid past a vertical moving permeable plate has been investigated with thermal radiation and heat source in the presence of porous material. The set of governing equations have been solved analytically using perturbation technique. Based on the obtained results, the following conclusions are advanced:

1. The temperature and the velocity profiles are increased with increasing values of R and Z respectively.
2. The velocity profile is significantly influenced by increasing values of Darcy number.
3. Intensifying magnetic field strength leads to reduction of drag force at the plate thereby decelerating fluid flow.
4. The skin friction is enhanced with accelerated values of Da, R and Z while a reverse situation occurs for increasing values of magnetic field.
5. The rate of heat transfer is improved for higher values of R and Z while the opposite phenomenon is recorded for increasing values of Pr.

Conflict of interest

The authors declare no conflict of interest.

References

- [1] S. K. Ahmad , S. Abdulsalam and M. M. Hamza , “Free convection slip flow with diffusion-thermo in a convectively heated vertical porous channel,”. *Journal of Applied Physical Sciences International*, vol. 8, p.58-71,2017.
- [2] K. Javaherdeh, K., Mirzaei, N. M. and M. Moslemi, “Natural Convection Heat and Mass Transfer in MHD Fluid Flow Past a Moving Vertical Plate with Variable Surface Temperature and Concentration in a Porous Medium,” *Engineering Science and Technology an International Journal*, vol. 18: p. 423-431,2015.
- [3] V. Vanita and A. Kumar “Effect of Radial Magnetic Field on Free Convective Flow over Ramped Velocity Moving Vertical Cylinder with Ramped Type Temperature and Concentration,” *Journal of Applied Fluid Mechanics*, vol. 9,p. 2855-2864 2016.
- [4] R. Muthuraj and S. Srinivas “Mixed convective heat and mass transfer in a vertical wavy channel with travelling thermal waves and porous medium,” *Journal of computers and mathematics with applications*, vol. 59, p. 3516-3528, 2010.

- [5] W. T. Cheng and C. H. Lin, "Transient mixed convective heat transfer with melting effect from the vertical plate in a liquid saturated porous medium," *International Journal of Engineering Science*, vol. 44, p. 1023–1036, 2006.
- [6] A. Postelnicu, "Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects," *International Journal of Heat and Mass Transfer*, vol. 47, p. 1467–1472, 2004.
- [7] M. B. K. Moorthy and K. Senthilvadivu, "Soret and Dufour effects on natural convection flow past a vertical surface in a porous medium with variable viscosity," *Journal of Applied Mathematics*, vol., Article ID 634806, 15 pages, 2012.
- [8] M. Guria and R.N. Jana, "Hydrodynamic flows through vertical wavy channel with traveling thermal waves embedded in porous medium," *International Journal of Application Mechanics Engineering*, vol. 3, p. 609-621, 2006.
- [9] A. Bahloul, M. A. Yahiaoui, P. Vasseur, R. Bennacer and H. Beji, "Natural convection of a two-component fluid in porous media bounded by tall concentric vertical cylinders," *Journal of Applied Mechanics*, vol. 73, p. 26–33, 2006.
- [10] C-Y Cheng, "Soret and Dufour effects on free convection boundary layer over a vertical cylinder in a saturated porous medium.," *International Journal of Communication in Heat and Mass Transfer*, vol. 37, p. 796–800, 2010.
- [11] C-Y Cheng, "Soret and Dufour effects on free convection heat and mass transfer from an arbitrarily inclined plate in a porous medium with constant wall temperature and concentration," *International Journal of Communication in Heat and Mass Transfer*, vol. 39, no. 1, p. 72–80, 2012.
- [12] W. J. Chang and D. F. Yang, "Natural convection for the melting of ice in porous media in a rectangular enclosure," *International Journal of Heat and Mass Transfer*, vol. 39, p. 2333–2348, 1996.
- [13] M. A. Mansur, "Radiation and free-convection effect on the oscillatory flow past a vertical plate," *NASA astrophysics data system, Kluwer Academic Publishers*, p.269–275, 1989.
- [14] M. M. Hamza, "Free convection slip flow of an exothermic fluid in a convectively heated vertical channel," *Ain Shams Engineering Journal*, <http://dx.doi.org/10.1016/j.asej.2016.08.011>.
- [15] B. K. Jha, B. Y. Isah and I. J. Uwanta, "Combined effect of suction/injection on MHD free-convection flow in a vertical channel with thermal radiation," *Ain Shams Engineering Journal*, DOI: 10.1016/j.asej.2016.06.001.
- [16] O. D. Makinde, "Free convection flow with thermal radiation and mass transfer past a moving vertical porous plate," *International Communication of Heat and Mass Transfer*, vol. 32, p.1411–9, 2005.
- [17] B. K. Jha, B.Y. Isah and I. J. Uwanta, "Unsteady MHD free convective Couette flow between vertical porous plates with thermal radiation," *Journal of King Saud University - Science* doi: <http://dx.doi.org/10.1016/j.jksus.2015.06.005>.
- [18] S. Shateyi, "Thermal radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing," *Journal of Applied Mathematics*, vol. Article ID 414830, 12 pages, 2008.
- [19] M. Narahari, "Effects thermal radiation and free convection currents on the unsteady couette flow between vertical parallel plates with constant heat flux at one boundary," *WSEAS Transactions on heat and mass transfer*, vol. 5, p. 21-30, 2010.
- [20] R. K. Deka and A. Bhattacharya, "Unsteady free couette flow of heat generating/absorbing fluids in porous medium," *International Journal of Mechanical Archives*, vol. 2, p. 853-863, 2011.
- [21] A. G. V. Kumar and S. V. K. Varma, "Radiation effects on MHD flow past an impulsively started exponentially accelerated vertical plate in variable temperature in the presence of heat generation," *International Journal of Engineering Science and Technology*, vol. 3, p. 2897-2909, 2011.
- [22] H. Mozayyeni and A. B. Rahimi, "Mixed convection in cylindrical annulus with rotating outer cylinder and constant magnetic field with an effect in radial direction," *Journal of Science Iranica*, vol. 19, p. 91–105, 2012.
- [23] M. Sheikholeslami, I. Hashmi and S. Soleimani, "Numerical investigation of the effect of magnetic field on natural convection in a curved shape enclosure. *Mathematical Problems in Engineering* .dx.doi.org/10.1155/2013/831725.
- [24] Singh R. K., and Singh A.K. Effect of induced magnetic field on natural convection in vertical concentric annuli. *Acta Mechanical Sinica*, vol. 28, p. 315-323, 2012.
- [25] B. K. Jha, R. A. Chia and A. Babatunde, "Natural convection flow in a vertical concentric annuli filled with porous material

having variable porosity under radial magnetic field: an exact solution," *International of Journal of Applied Physical Science*, vol. 4, p. 2395-5279, 2015.

Nomenclature

m^2 = Magnetic field effect

Da = Darcy number

θ_0 = Initial temperature of the fluid

θ = Dimensionless temperature of the fluid

β_0 = Coefficient of magnetic field expansion

R = Thermal radiation parameter

Z = Heat source parameter

σ = Electrical conductivity of the fluid

α = Thermal diffusivity

ρ = Density of the fluid

β = Coefficient of thermal expansion

μ_e = Magnetic permeability

K_0 = Constant permeability of the porous

medium

ν = Kinematic viscosity of the fluid

g = Acceleration due to gravity

Nu = Nusselt number

τ = Skin friction

Appendix

$$P = \left(m^2 + \frac{1}{K} \right), m_1 = \frac{s \text{Pr} + \text{sgn} \left((s \text{Pr})^2 - 4Z \right)}{2}, m_2 = \frac{-s \text{Pr} + \text{sgn} \left((s \text{Pr})^2 - 4Z \right)}{2}, m_3 = \frac{s + \text{sgn} \left((s)^2 + 4P \right)}{2},$$

$$m_4 = \frac{-s + \text{sgn} \left((s)^2 + 4P \right)}{2}, A_1 = \frac{m_2 A_2 - 1}{m_1}, A_2 = \frac{\exp(m_1)}{m_2 \exp(m_1) + m_1 \exp(-m_2)}, A_3 = \frac{m_2 (A_4 + D_2) - m_1 D_1 - 1}{m_1}$$

$$A_4 = \frac{\exp(m_1) - m_2 D_2 \exp(m_1) + m_1 D_2 \exp(-m_2)}{m_2 \exp(m_1) + m_1 \exp(-m_2)}, A_5 = L_\rho - A_6 - D_3 - D_4,$$

$$A_6 = \frac{D_3 (\exp(m_3) - \exp(m_1)) + D_4 (\exp(m_3) - \exp(m_1)) - L_\rho \exp(m_3)}{\exp(-m_4) - \exp(m_1)}, A_7 = L_\rho - A_8 - D_5 - D_6,$$

$$A_8 = \frac{D_5 (\exp(m_3) - \exp(m_1)) + D_6 (\exp(m_3) - \exp(m_1)) - L_\rho \exp(m_3)}{\exp(-m_4) - \exp(m_1)}, D_1 = A_1 (m_1^2 - sm_1 \text{Pr} + Z),$$

$$D_2 = A_2 (m_2^2 + sm_2 \text{Pr} + Z), D_3 = \frac{-(m_1^2 A_4 K - sm_1 A_4 - m^2 A_4 K - A_1)}{K}, D_4 = \frac{-(m_2^2 A_2 K - sm_2 A_2 - m^2 A_2 K - A_2)}{K},$$

$$D_5 = \frac{-(m_1^2 (A_3 + D_1) K - sm_1 (A_3 + D_1) K - m^2 (A_3 + D_1) K - (A_3 + D_1))}{K},$$

$$D_6 = \frac{-(m_1^2 (A_4 + D_2) K - sm_1 (A_4 + D_2) K - m^2 (A_4 + D_2) K - (A_4 + D_2))}{K},$$