# **Caliphate Journal of Science & Technology (CaJoST)**



#### **Article Info**

Received: 17<sup>th</sup> February 2020 Revised: 2<sup>nd</sup> April 2020 Accepted: 6<sup>th</sup> April 2020

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**Cite this:** *CaJoST,* 2020, 2, 73-80

# **Effect of Radiation on Heat and Mass Transfer Magnetohydrodynamic Casson Fluid Flow in a Vertical Channel**

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The present study examines effect of radiation on Magnetohydrodynamic (MHD) convective boundary layer, flow of a Casson fluid with heat and mass transfer in a channel. The fluid investigated is electrically conducting. The governing flow equations are converted into dimensionless form using appropriate non-dimensional parameters and variables and then numerically solved by explicit finite difference method (EFDM). Special attention was given to analyse the effect of thermo physical parameters of engineering interests associated with the flow quantities viz. velocity, temperature, concentration field and presented through graphs after series of numerical experiments. It was found that heat generation has tendency to enhance the fluid velocity as an opposite result is seen with chemical reaction parameter. Comparison of present results with the previous literature was carried out to show the accuracy of the results.

**Keywords:** Casson fluid; radiation; MHD; EFDM; heat and mass transfer.

# **1. Introduction**

The study of flow of electrically conducting fluid past a porous plate is given primary importance due to its numerous applications in Science and Engineering field like magnetohydrodynamics (MHD) generators, nuclear reactors, oil exploration, boundary layer control in the field of aerodynamics, plasma studies, geothermal energy extraction, solar collectors, jet nozzles, turbine blades and heat exchangers. Specifically, they are greatly used to insulate a heated body to maintain its temperature.

When technological and engineering processes occur at high temperature. Then, the knowledge of thermal radiation heat transfer becomes very important and its effect cannot be neglected. The quality of final products depend greatly on heat controlling factors and knowledge of the effect of thermal radiation therefore is useful industrially for the design of reliable and desired equipment and products. Examples include nuclear power plant, solar power technology, steel industry, fossil fuel combustion, space science etc. Vedavathi *et al.* [1] studied the effects of heat and mass transfer on two- dimensional unsteady MHD convection flow past a vertical porous plate in the presence of thermal radiation under the influence of Dufour and Soret effects also (Grief *et al.* [2]); Hossain *et al.* [3]; Youn [4]; Srinivas and Muthuraj [5]) are few amongst authors who have conducted studies on thermal radiation.

Convection is of three forms which are force, mixed and free. Free convection is important amongst them due to its presence in engineering processes such as automatic control systems which consist of electronic and electrical components, constantly subjected to heating and cooled by means of free convection (Khalid *et al.* [6]). The need to investigate the effect of radiation on unsteady free convection flow on heat and mass transfer problem has become very important. This is because at high operating temperature, radiation effect can be quite significant. Many manufacturing, industrial and engineering processes occur at high temperature and knowledge of radiation heat transfer becomes necessary for design of reliable equipment.

Heat and mass transfer plays an important role in manufacturing and industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, space craft design, satellites and space vehicles. The usefulness of heat and mass transfer can also be explored in our environment, in drying, filtration processes, saturation of porous materials by chemicals, solar energy collectors, nuclear reactors Kateria and Petal [7]. Few authors who carried out studies on heat an mass transfer are; Uwanta and Hamza [8] investigated transient heat transfer flow in a vertical channel with porous medium and periodic suction analytically by perturbation method. Asogwa *et al.* [9] examined heat and mass transfer over a vertical plate with periodic suction and heat sink.

In nature, some non-Newtonian fluids exist like elastic solid. Casson fluid is an example of such fluids with unique characteristics. Few examples of Casson fluid are jelly, tomato sauce, honey, liquid soup and concentrated fruit juices. Human blood is also a Casson fluid due to the blood cells' chain structure and contents present in aqueous base plasma like rouleaux, fibrinogen, globulin and protein. Casson model was introduced by Casson in 1959 for the prediction of the flow behavior of pigment-oil suspensions (Casson, [10]). Some studies containing the Casson fluid phenomenon are found in the works of (Hayat *et al.* [11]; Ajayi *et al.* [12]; Raju *et al.* [13]; Rajakumar *et al.* [14], Omokhuale and Altine [15]). Explicit Finite difference method (EFDM) is very useful in solving coupled nonlinear partial differential equations whose solutions cannot be obtained analytically. The unsteady natural convection flow past a semiinfinite vertical plate was first solved by Hellum and Churchill [16] using explicit finite difference method. Few researchers who have employed the EFDM are (Sharma *et al.* [17]; Reddy and Janardhan *et al.* [18]; Ahmed and Alam [19]). Recently, Mondal *et al.* [20] conducted a numerical exploration on inscribed MHD unsteady heat and mass transfer of Casson nanofluid flow where variable thermal conductivity, radiation and heat absorption are counterfeited using EFDM. The effect of heat generation on boundary layer fluid flow is very important because of its usefulness in engineering such as radioactive materials, fire and combustion, metal waste, reactor safety analysis, spent nuclear fuel etc. Some scholars who have researched on heat generation with respect to several phenomenon are (Jasmine *et al.* [21]; Animasaun *et al.* [22], Mohan *et al.* [23]). Omokhuale and Jabaka [24] studied magnetohydrodynamic Casson Fluid Flow over an Infinite Vertical Plate with Chemical Reaction and Heat Generation numerically using EFDM.

The solutions of Casson fluid were mostly obtained in closed form using Laplace technique in all the studies mentioned so far. There are few instances in which the numerical schemes were employed. To the best of the authors' knowledge, no investigation has been reported yet to study the effects of radiation on heat and mass transfer MHD Casson fluid flow in a

vertical channel numerically using EFDM. The present work aims to fill the gap that exists in literatures.

# **2. Mathematical Formulation**

We consider Casson fluid over an infinite vertical plate embedded in a saturated porous medium.

The flow being restricted to  $y > 0$  , where  $y$  is the coordinate is be measured in the normal direction to the plate. The fluid is assumed to be electrically conducting with a uniform magnetic

field  $\, B \,$  of strength  $B_{0}^{}$  , applied in a direction perpendicular to the plate. The plate temperature

is raised to  $^{\pmb{T}_w}$  which is thereafter be maintained constant.

The rheological equation of state for the Cauchy stress tensor of Casson fluid is given by,

$$
\tau = \tau_0 + \mu \tau^*
$$

$$
\tau_0 = \begin{cases} 2\left(\mu_B + \frac{P_y}{\sqrt{2\pi}}\right) e_{ij}, \text{ when } \pi > \pi_c \\ 2\left(\mu_B + \frac{P_y}{\sqrt{2\pi_c}}\right) e_{ij}, \text{ when } \pi < \pi_c \end{cases}
$$

where

where 
$$
\pi = e_{ij}e_{ij}
$$
 and  $e_{ij}$  is the  $(i, j)^{th}$ 

component of the deformation rate.

The following assumptions were made before deriving the governing equations; incompressible flow, free convection and non-Newtonian flow, infinite vertical plate, fluid suction and magnetic field are imposed at the plate surface, the temperature and concentration of the fluid are raised to  $T'_w$  and  $C'_w$  respectively and are higher

than the ambient temperature and that of fluid. In addition, the effects of radiation, thermal conductivity, chemical absorption is taken into account. It is also be assumed that the viscous dissipation is present.

The governing equations of the flow under the usual Boussinesq and boundary-layer approximation can be written as:

$$
\frac{\partial v'}{\partial y'} = 0\tag{1}
$$

$$
\rho \left( \frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} \right) = \mu_B \left( 1 + \frac{1}{\gamma} \right) \frac{\partial^2 u'}{\partial y'^2} - \sigma B_0^2 u' - \frac{\mu \eta}{K_1} u'
$$

+
$$
g \rho \left[ \beta_r (T' - T'_{\infty}) + \beta_m (C' - C'_{\infty}) \right]
$$
  
\n+ $g \rho \left[ \frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} \right] = k_0 \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} - Q(T' - T'_{\infty}) + v \left( \frac{\partial u'}{\partial y'} \right)^2$   
\n $\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - R^* (C' - C'_{\infty})$   
\n(4)

with the following initial and boundary conditions:

$$
t' \le 0, u' = 0, T' = T'_0, C' = C'_0 \text{ for all } y'
$$
  
\n
$$
t' > 0, u' = 0, T' = T'_w, C' = C'_w \text{ at } y' = 0
$$
  
\n
$$
u' = 0, T' = T'_0, C' = C'_0 \text{ at } y' = h
$$
  
\n(5)

where  $u$  and  $v$  are velocity components in  $x'$  and *y* directions respectively, *T* is the temperature, *t* is the time, g is the acceleration due to gravity,  $\beta$ is the thermal expansion coefficient,  $\overline{\beta}^*$ is the concentration expansion coefficient,  $\mathcal{V}$  is the kinematic viscosity,  $D$  is the chemical molecular diffusivity, *Cp* is heat capacity at constant pressure,  $B_0$  is a constant magnetic field intensity,  $\sigma$  is the electrical conductivity of the fluid,  ${}^{k_0}$  is the variable thermal conductivity,  ${}^{\!\mathcal{P}}$ is the density,  $\overline{\mathcal{X}}_1$  is the Jeffery fluid,  $\mathcal{T}_w$  is the wall temperature,  $T_{\infty}$  is the free stream temperature,  $\,C_{^w}$  is the species concentration at the plate surface,  $\mathit{C}_{\infty}$  is the free stream  $\overline{\mathcal{O}}$  is the heat generation coefficient.

 $v_0 > 0$  is the suction parameter and  $v_0 < 0$  is the injection parameter.

For the case of an optically thin gray gas the local radiant absorption is expressed by

$$
\frac{\partial q_r}{\partial y'} = -4a_R \sigma \left( T'^4_{\infty} - T'^4 \right)
$$
\n(6)

we assume that the temperature difference within the flow are sufficiently small such that  $T'^4$  may be expressed as a linear function of the temperature. This is accomplished by expanding  $T'^4$  in Taylor series about  $T'_{\infty}$  and neglecting higher-order terms, thus

$$
T^{\prime 4} \cong 4T^{\prime 3}_{\infty}T^{\prime} - 3T^{\prime 4}_{\infty} \tag{7}
$$

By using equations (6) and (7), equation (2) reduces to

$$
\left[\frac{\partial T'}{\partial t'} + v \frac{\partial T'}{\partial y'}\right] = \frac{k_o}{\rho C p} \frac{\partial^2 T'}{\partial y'^2} - \frac{16 a_R \sigma T_w^3}{\rho C p} \left(T' - T_w'\right) - \frac{Q}{\rho C p} \left(T' - T_w'\right) + v \left(\frac{\partial u'}{\partial y'}\right)^2
$$
\n(8)

On introducing the following non-dimensionless quantities

$$
y = \frac{u_0}{v} y', u_p = \frac{u'_p}{u_0}, t = \frac{u_0^2}{v} t', u = \frac{u'}{u_0}, \theta = \frac{T' - T'_\infty}{T_w - T_\infty}, \phi = \frac{C' - C'_\infty}{C_w - C_\infty}, \tau^* = \frac{\tau}{\rho u^2}\n\}
$$
(9)

Using  $(1)$  and  $(9)$ , equations  $(3)$  -  $(5)$  and  $(8)$ become:

$$
\frac{\partial u}{\partial t} - \lambda \frac{\partial u}{\partial y} = \left(1 + \frac{1}{\gamma}\right) \frac{\partial^2 u}{\partial y^2} - M^2 u - \frac{1}{K} u + Gr\theta + GcC
$$
\n(10)

$$
\frac{\partial \theta}{\partial t} - \lambda \frac{\partial \theta}{\partial y} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} - \frac{R}{\text{Pr}} \theta - \psi \theta + Ec \left(\frac{\partial u}{\partial y}\right)^2
$$
\n(11)

$$
\frac{\partial \phi}{\partial t} - \lambda \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - \chi \phi \tag{12}
$$

The corresponding boundary conditions are:

$$
u = 0, \theta = 0, \phi = 0 \text{ for all } y \neq 0
$$
  
\n
$$
u = u_p, \theta = 1, \phi = 1 \text{ at } y = 0
$$
  
\n
$$
u = 0, \to \theta = 0, \phi = 0 \text{ as } y = 1
$$
  
\n(13)

where  $\mathscr Y$  is the Casson parameter,  $\,Gr$  is the thermal Grashof number,  $\emph{Gc}\,$  is the mass

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Grashof number,  $Sc$  is the Schmidt number, Pr is the Prandtl number, *M* is the magnetic parameter,  $\chi$  is the chemical reaction parameter,  $\,K\,$  is the permeability parameter,  $\lambda$  is the suction parameter,  $\mathscr V$  is the heat generation parameter,  $\mathit{Ec}\,$  is the Eckert number.

# **3. Calculation Technique**

To get the difference equations, the region of the flow is divided into a grid or mesh of line parallel to  $\overline{y}$  axis. Where  $\overline{y}$ -axis is normal to the plate. It is considered that the plate is of height  $y_{\text{max}} = 150$  i.e. *y* varies from 1 to 150. There are  $m = 150$  and  $n = 300$  grid spacing in the <sup>y</sup>-directions. It is assumed that  $\Delta y$  is a constant mesh size along  $y$  direction and taken  $\frac{1}{2} \Delta y = 0.50 (0 \le y \le 150).$ with the smaller time-step,  $\Delta t = 0.0005$  and  $u', T'$  and  $C'$ represents the values of  $^{u,\theta}$  and  $^{C}$  at the end of a time step respectively.

Therefore, the nonlinear coupled partial differential equations (7) to (9) subject to boundary conditions (10) are solved numerically

using explicit finite difference method as below:  
\n
$$
\left(\frac{u'_{i,j} - u_{i,j}}{\Delta t} - \lambda \frac{u_{i,j+1} - u_{i,j}}{\Delta y}\right) = \left(1 + \frac{1}{\gamma}\right) \left[\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{(\Delta y)^2}\right]
$$

$$
-M(u_{i,j}) - \frac{1}{K}(u_{i,j}) + Gr(\theta_{i,j}) + Gc(C_{i,j})
$$
\n(11)

(11)  
\n
$$
\left(\frac{\theta'_{i,j+1} - \theta_{i,j}}{\Delta t} - \lambda \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta y}\right) = \frac{1}{\text{Pr}} \left[\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta y)^2}\right] - \frac{R}{\text{Pr}}(\theta_{i,j+1} + \theta_{i,j})
$$

$$
-\psi \left(\theta_{i,j+1} + \theta_{i,j}\right) + Ec\left(\frac{u_{i,j+1} - u_{i,j}}{\Delta y}\right)^2
$$
\n
$$
Sc\left(\frac{\phi'_{i,j} - \phi_{i,j}}{\Delta t} - \lambda \frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta y}\right) = \left[\frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{\left(\Delta y\right)^2}\right] - \frac{\chi Sc}{2}(\phi_{i,j})
$$
\n(13)

The initial and boundary conditions becomes

$$
u_{i,0} = 0, \theta_{i,0} = 0, \phi_{i,0} = 0 \text{ for all i except } i = 0
$$
  

$$
u_{i,0} = u_p, \theta_{i,0} = 1, \phi_{i,0} = 1
$$
  

$$
u_{L,0} = 0, \theta_{L,0} = 1, \phi_{L,0} = 1
$$
 (14)

where L corresponds to  $\infty$ . The suffix i corresponds to y and j is equals to t. consequently,  $\Delta t = t_{j+1} - t_j$  and  $\Delta y = y_{i+1} - y_i$ .

The finite difference approximations of these equations were solved by using the values for Gr  $=$  Gc = M =  $\lambda$  = 1,  $\gamma$  = 0.5,  $\lambda$  = 0.5,  $Pr$  = 0.71,  $Sc = 0.6$ , S = 0.2, Ec = 0.2,  $\chi$  = 0.5, K = 0.5, except where they are varied. A step size of  $\Delta y$  = 0.01 is used for the interval  $y_{\text{min}} = 0$  to  $y_{\text{max}} = 5$  for a desired accuracy and a

convergence criterion of  $10^{-6}$  is satisfied for various parameters.

#### **4. Results and Discussion**

An explicit finite difference technique has been adopted to obtain the numerical results of the present model has been carried out for the different values of parameters embedded on the velocity, temperature and concentration within the boundary conditions. The finite difference approximations of these equations were solved by using the values for Gr = Gc = M =  $\lambda$  = 1,  $\gamma = 0.5$ ,  $Pr = 0.71$ ,  $Sc = 0.6$ ,  $S = 0.2$ ,  $Ec = 0.2$ ,  $\lambda$  = 0.5, K= 0.5, except where they are varied. The interval  $y_{min} = 0$  to  $y_{max} = 5$  for a desired

accuracy and a convergence for various parameters.

#### **4.1 Velocity profiles**

Figures 1 to 7 depict the velocity profiles with varying parameters respectively.

Figure 1 shows the effect of Prandtl number on the velocity. It is seen that, the velocity decreases with increasing Prandtl number. Effect of Hartmann number M on the velocity is depicted in Figure 2, it is observed that, the velocity decreases with the increase in magnetic parameter. This is due to the fact that application of transverse magnetic field to an electrically conducting fluid which results in a Lorentz force (a resistive type force) similar to the drag force which leads to the deceleration of the flow. Casson parameter effect on the velocity profile is

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illustrated in Figure 3. It is found that, the velocity decreases with increasing Casson parameter. It is interesting to note that Casson parameter makes the velocity boundary layer thickness shorter. This occurs because of the plasticity of Casson fluid. Effect of suction parameter on the velocity is presented in Figure 4. It is observed that, the velocity is lower due to an increase in suction parameter. Figure 5 shows varied values of thermal Grashof number on the velocity, it is noticed that, the velocity rises with increasing thermal Grashof number. In Figure 6, the effect of mass Grashof number on the velocity is presented. It is seen that, the velocity increases with increase in mass Grashof number. Buoyancy force acts like a favourable pressure gradient which accelerates the fluid within the boundary layer. The variation of radiation parameter with respect to temperature profile is shown in Figure 7. The increment in radiation parameter increases the velocity of the fluid. This is due to the fact that heat flux of surface increases which leads velocity profile to grow up.



Figure 1. Effect of Prandtl number on velocity.



Figure 2. Effect of Hartmann number on velocity.



Figure 3. Effect of Casson parameter on velocity.







Figure 5. Effect of thermal Grashof number on velocity.





Figure 8. Effect of Prandtl number on Temperature.



Figure 7. Effect of radiation on velocity.

#### **4.2 Temperature profiles**

Figures 8 to 9 demonstrate the temperature profiles.

In Figure 8, the influence of Prandtl number on the temperature is shown. It is seen that, the temperature decreases when the Prandtl number is increased. It is justified due to the fact that thermal conductivity of the fluid decreases with higher Prandtl number and hence retards the thermal boundary layer thickness. Figure 9 represents effect of radiation parameter on temperature. It is observed that, the temperature increases with increase in radiation.

## **4.3 Concentration profiles**

Figures 10 to 11 display the concentration profiles.

Effect of Schmidt number with respect to concentration are visualized in Figure 10. As Schmidt number is the ratio of viscous diffusion rate to mass diffusion rate. Hence, the growth in Schmidt number declines the concentration of the fluid. Figure 11 shows the influence of chemical reaction parameter on the concentration. It is demonstrated that. The concentration is lower as the chemical reaction parameter is increased.



Figure 10. Effect of Schmidt number on Concentration.



Figure 11. Effect of chemical reaction parameter on Concentration.

# **Acknowledgement**

The authors are grateful to Tertiary Education Trust Fund (TETFUND), Abuja, Nigeria for funding this research work under the Institutional

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# **5. Conclusion**

Radiation on Magnetohydrodynamic (MHD) convective boundary layer flow of a Casson fluid with heat and mass transfer in a channel.. The flow was governed by a modeled coupled nonlinear system of partial differential equations (PDEs) in dimensional form which are transformed into non-dimensional form using some non-dimensional variables. EFDM was used to approximate the velocity, temperature, concentration field. Our findings from the current study are:

- 1. Velocity profile decreases with increasing values of Casson parameter.
- 2. Concentration boundary layer thickness reduces by increasing the values of Schmidt number and increases by increasing the chemical reaction parameter.
- 3. Magnetic field retards the motion of the fluid within the boundary layer. Similarly, the velocity of the fluid rises with increasing values of thermal Grashof and mass Grashof numbers.
- 4. Momentum and thermal boundary layer thickness of the fluid increases as<br>radiation becomes significant becomes significant

#### Based **Research**. **(TETFUND/DRSS/UNI/GUSAU/2015/RPVOL.1)**.

# **Conflict of interest**

The authors declare no conflict of interest.

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