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Magnetized Thermal Boundary Layer Flow over a Vertical Plate under the Influence of Convective Boundary Conditions

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This study examines the influence of thermal buoyancy on laminar boundary layer flow and heat transfer around a vertical plate immersed in a uniform fluid stream. The governing equations, which are nonlinear partial differential equations, are transformed into a system of coupled nonlinear ordinary differential equations by introducing a similarity variable. The resulting equations are then solved numerically using the shooting method combined with a fourth-order Runge-Kutta integration scheme. The effects of key flow parameters, including the Prandtl number (Pr), local Grashof number (Gr), and convective heat transfer parameter (Bi), are analyzed and presented both graphically and in tabular form. The influence of these parameters is further illustrated through velocity and temperature profiles. The findings are validated by comparing them with previously published results, showing excellent agreement. The analysis demonstrates that thermal buoyancy plays a significant role in shaping boundary layer flow and heat transfer dynamics. Additionally, the Prandtl number, Grashof number, and Biot number are found to have a notable impact on surface temperature and the characteristics of the fluid-solid interface.

Keywords: Buoyancy, Boundary Layer Flow, Heat Transfer, Thermal and Runge-Kutta.

1. Introduction

Convective heat transfer is essential for optimizing thermal management systems, especially in high-temperature environments. This study addresses a significant gap in knowledge by exploring the combined effects of buoyancy forces and convective heat transfer on laminar boundary layer flow. Effective heat transfer is critical in high-temperature applications, including gas turbines, nuclear power plants, thermal energy systems, and aerospace engineering. These industries depend on efficient thermal management to ensure safety, performance, and energy efficiency. This research delves into the influence of buoyancy forces on fluid dynamics and heat exchange, providing insights into the intricate relationship between convective heat transfer and laminar boundary layer flow in high-temperature scenarios. . Bachok et al. (2010) examined Flow and heat transfer at a general three-dimensional stagnation point in a nanofluid. Grosan and Pop (2011) studied Axisymmetric mixed convection boundary layer flow past a vertical cylinder in a nanofluid

Historically, Blasius (1908) was the first to solve the classical problem of fluid flow along stationary vertical and horizontal surfaces in a uniform freestream. Recent studies, such as those by Abdullahi and Zayyanu (2024), have explored heat and mass transfer in mixed convection flow within vertical channels. Bachok et al (2012) investigated

Flow and heat transfer characteristics on a moving plate in a nanofluid. Fenuga et al. (2018) analyzed thermal boundary layer flow over a vertical plate, considering electrical conductivity and convective surface boundary conditions. Yacob et al (2011) investigated Falkner–Skan problem for a static or moving wedge in nanofluids. Wey (1942) investigated the differential equations governing simple boundary-layer problems, while Seun et al. (2023) studied the effects of thermal radiation on non-Newtonian Casson fluid flow through porous materials under magnetic fields and buoyancy forces. Magyari (2008) conducted experimental and empirical research on the moving plate thermometer, and Cortel (2005) used numerical methods to solve the classical Blasius flat plate problem. He (2003) provided an analytical solution using a perturbation method, and Bataller (2008) numerically examined the combined effects of convective heat transfer and thermal radiation on laminar boundary layer flow over a flat plate. Alveroglu et al. (2016) studied The effect of surface roughness on the convective instability of the BEK family of boundary-layer flows. Aziz (2008) studied laminar thermal boundary layers over flat plates under specific boundary conditions. Al-Malki (2021) investigated The effects of roughness levels on the instability of the boundary-layer flow over a rotating disk with an enforced axial flow. Makinde and

Sibanda (2008) investigated mixed convective magnetohydrodynamic flow, heat, and mass transfer over a vertical plate in a porous medium, highlighting the role of wall suction. Makinde (2009) also analyzed reactive non-Newtonian flow in cylindrical pipes, and Cortell (2008) explored similarity solutions for flow and heat transfer over nonlinearly stretching surfaces. Belyaeva et al. (2024) investigated Acoustic Radiation of a Turbulent Boundary Layer Over a Flat Smooth Boundary. Wang et al. (2023) investigated Melting rheology of three-dimensional Maxwell Nanofluid (Graphene-Engine-Oil) flow with slip condition past a stretching surface through Darcy-Forchheimer medium. Wang et al. (2021) investigated Unsteady thermal transport flow of Casson nanofluids with generalized Mittag-Leffler kernel of Prabhakar's type

In this study, the works of Aziz (2009) and Makinde and Olarewaju (2008) are extended to incorporate the effects of buoyancy forces, thermal radiation, magnetic fields, and heat absorption. The problem is analyzed under convective boundary conditions. The governing highly nonlinear partial differential equations are transformed into ordinary differential equations using a similarity variable. These equations are then solved numerically using the shooting method and a sixth-order Runge-Kutta method. The coding and simulations are performed using Maple mathematical software. The effects of key flow parameters on fluid dynamics and temperature distribution are presented graphically and in tabular form. A detailed analysis and discussion of these parameters are provided, and the results are validated against existing literature.

2. Problem Formulation

The study focuses on a two-dimensional, steady, incompressible fluid flow combined with heat transfer over a stationary vertical plate. The analysis includes the effects of heat absorption and thermal radiation. On the right side of the plate, a cold fluid stream flows uniformly at a constant temperature and velocity. Meanwhile, the left surface of the plate is heated convectively by a hot fluid at a specific temperature, characterized by a given heat transfer coefficient. A magnetic field is applied perpendicular to the heated plate. The problem is formulated under convective boundary conditions. Using the Boussinesq approximation, the governing equations for continuity, momentum, and energy are derived to model the flow and temperature distribution.

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma \beta_0^2}{\rho} + \nu \frac{\partial^2 u}{\partial y^2} + u \frac{\partial^2 u}{\partial x^2} + g\beta(T - T_\infty) \quad (2)$$

Energy Equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \frac{\partial q_r}{\rho C_p} + \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_p} (T - T_\infty) \quad (3)$$

flux. The velocity boundary conditions at the plate can be written as,

$$u(x, 0) = v(x, 0) = 0 \quad (4)$$

$$u(x, \infty) = U_\infty \quad (5)$$

The boundary conditions at the plate surface and far into the cold fluid may be written as

$$-k \frac{\partial T}{\partial y}(x, 0) = h_f (T_f - T(x, 0)) \quad (6)$$

$$T(x, \infty) = T_\infty \quad (7)$$

Introducing a similarity variable η and dimensionless stream function $f(\eta)$ and temperature $\theta(\eta)$

$$\eta = y \sqrt{\frac{U_\infty}{\nu x}} = \frac{y}{x} \sqrt{\text{Re}_x}, \quad \frac{u}{U_\infty} f', \quad (8)$$

$$v = (\eta f' - f) \sqrt{\frac{U_\infty \nu}{4x}}, \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}$$

were prime symbol denoted differentiation with respect to η and $\text{Re}_x = \frac{U_\infty x}{\nu}$ is the local Reynolds number.

Applied the similarity variables in equation (8) in governing equations (1-3), then equations (1-3) reduces to

$$f''' + \frac{1}{2} f f'' + f(\theta - M) f' + M f'' + Gr \theta = 0 \quad (9)$$

$$\theta'' + \frac{1}{2} (\text{Pr} f' f \theta' + f f'') + Q f' \theta + \theta' - Ra \theta' = 0 \quad (10)$$

$$f(0) = f'(0) = 0, \quad (11)$$

$$\theta'(0) = Bi(1 - \theta(0))$$

$$f'(\infty) = 1, \quad \theta(\infty) = 0 \quad (12)$$

$$\left. \begin{aligned} Bi &= \frac{h_f}{k} \sqrt{\frac{\nu x}{U_\infty}}, \Pr = \frac{\nu}{\alpha}, \\ Ra &= \frac{16\sigma T_\infty^3}{3kC_p}, Q = \frac{\nu^2 Q_0}{\sigma\rho C_p}, \\ M &= \frac{\sigma\beta_0^2 x}{\rho U_0}, Gr = \frac{\nu x g \beta (T_f - T_\infty)}{U_\infty^2} \end{aligned} \right\} \quad (13)$$

To ensure the momentum and energy equations yield a similarity solution, the parameters must remain constant and not vary with x , as outlined in Eq. 13. To meet this requirement, the heat transfer coefficient was selected to be proportional to $h = cx^{m-1}$, and the thermal expansion coefficient was chosen to be proportional to $\beta = cx^{m-3}$. Following this, we assume:

$$\beta = cx^{m-1} \quad \text{and} \quad h = cx^{m-1} \quad (14)$$

where cc and mm are constants. Substituting Eq. (14) into Eq. (13), we obtain the following, where u and v represent the velocity components along the plate axis x and perpendicular to it y , respectively. Here, β is the thermal expansion coefficient, ν is the kinematic viscosity of the fluid, T is the fluid temperature, α is the thermal diffusivity, Q is the heat source, and h is the heat transfer coefficient.

$$Bi = \frac{c}{k} \sqrt{\frac{\nu}{U_\infty}}, \quad Gr = \frac{\nu m g (T_f - T_\infty)}{U_\infty^2} \quad (15)$$

Numerical Solutions

The system of coupled nonlinear equations, specifically Equation (9) and Equation (10), along with the associated boundary conditions given in Equation (11) and Equation (12), are solved numerically. The fourth-order Runge–Kutta method, combined with the shooting technique, is employed for this purpose. The coding and simulation of the problem are carried out using Maple software."

3. Results and Discussion

Numerical simulations were conducted for various thermophysical parameters influencing the fluid dynamics within the flow regime. The Grashof number represents scenarios involving cooling, while higher values correspond to regions of elevated temperature. Such cooling phenomena are commonly observed in engineering contexts, particularly in the thermal management of electronic devices and nuclear reactors. The current findings were compared with prior studies, demonstrating strong consistency and validation.

Numerical calculations have carried out for different values of the thermophysical parameters controlling the fluid dynamics in the flow regime. The Grashof number $Gr > 0$ corresponds to the cooling problem and the $Gr < 0$ respond to hot region of the flow. The cooling problem is often encountered in

engineering applications; for example, in the cooling of electronic components and nuclear reactors. Comparisons of the present results with previously work is performed and excellent agreement has been obtained.

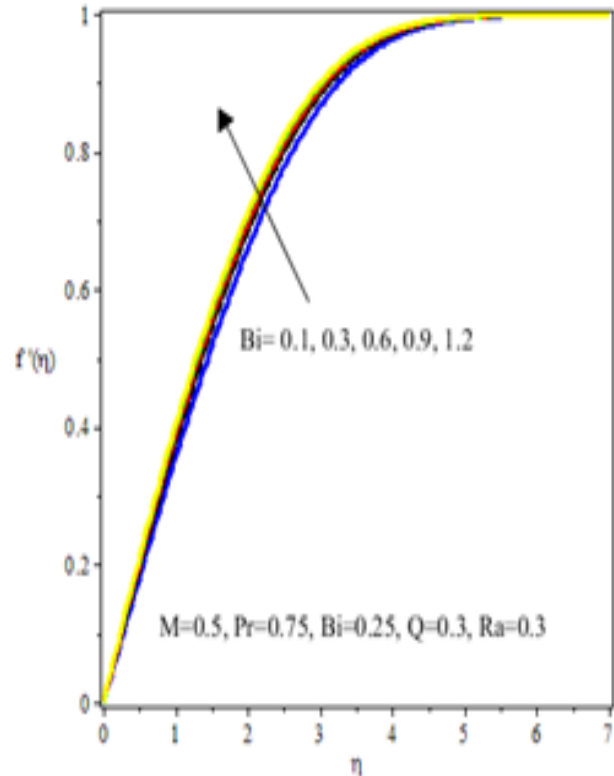


Fig 1: Influence of Bi on velocity profile.

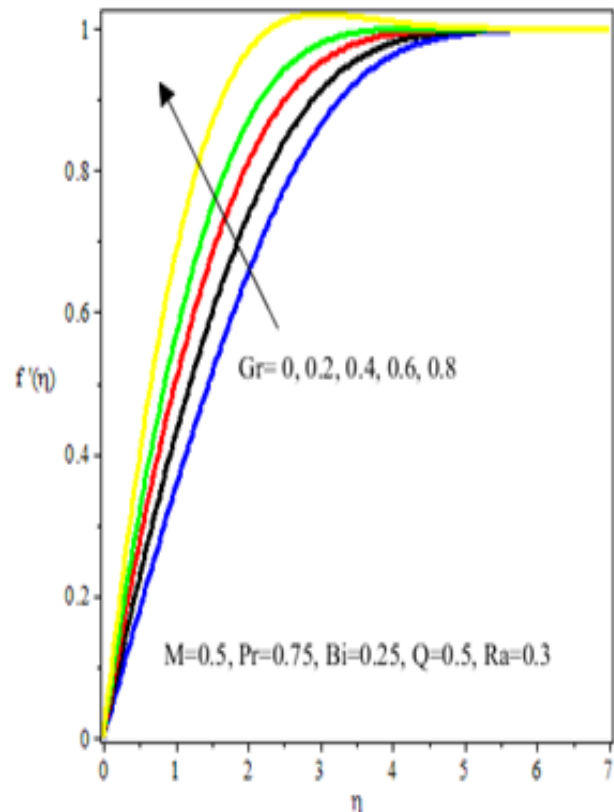


Fig 2: Influence of Gr on velocity profile

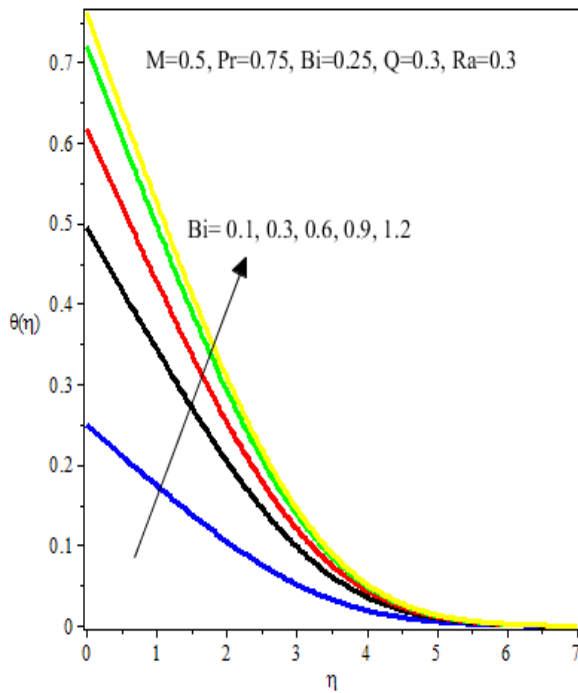


Fig 3: Influence of Bi on temperature profile

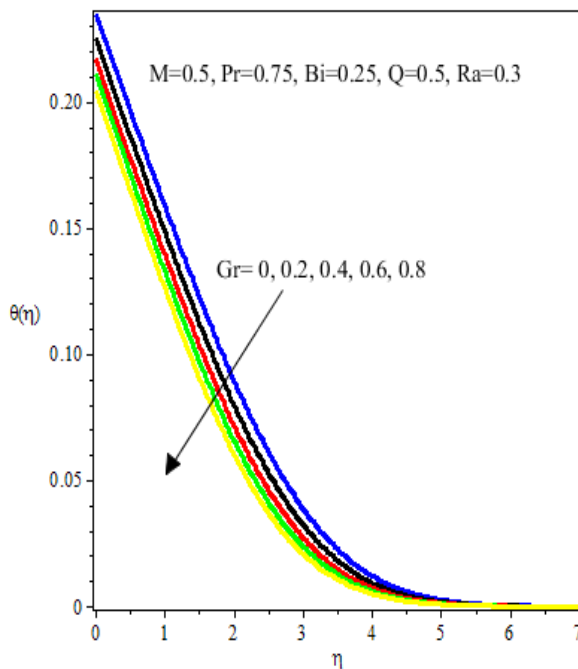


Fig 4: Influence of Gr on temperature profile

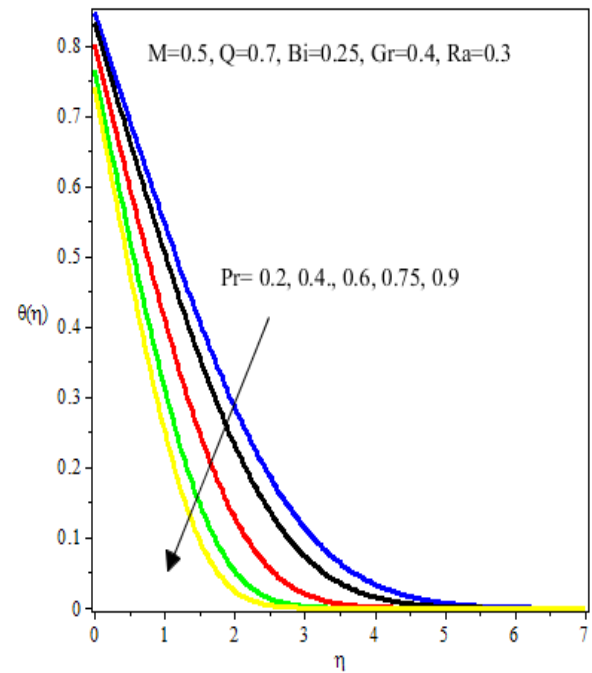


Fig 5: Influence of Pr on temperature profile

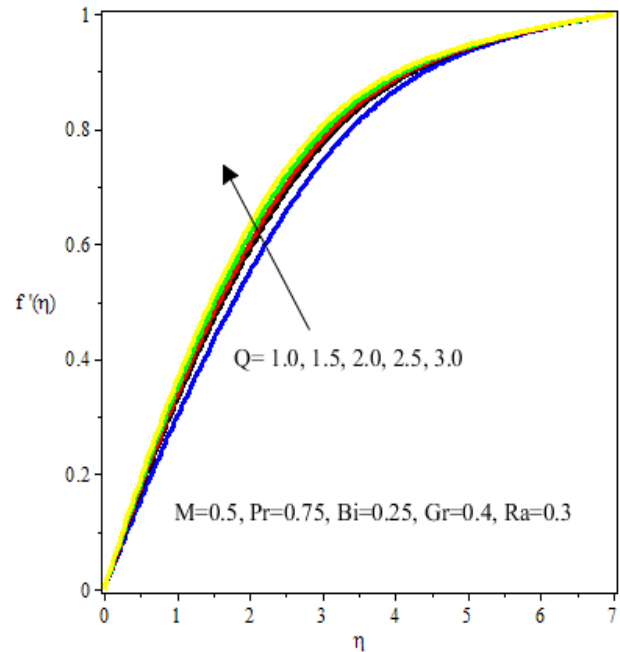


Fig 6: Influence of Q on velocity profile

Figures 1 and 2 illustrate the influence of the Biot number (Bi) and Grashof number (Gr) on the velocity distribution of the fluid. As expected, the fluid velocity is minimal at the plate surface and gradually rises as it moves away from the plate, eventually reaching the freestream value, which aligns with the boundary conditions. Notably, an increase in the convective heat transfer parameter (Bi) results in a slight enhancement of fluid velocity within the boundary layer. A comparable trend is observed when the local Grashof number increases, primarily due to buoyancy effects. However, the impact of the local Grashof number on velocity the

influence of the local Grashof number on the velocity profiles, as depicted in Fig. 3, is more significant compared to the impact of the convection parameter Bi shown in Fig. 1. Figs. 3–5 illustrate the temperature distribution of the fluid within the boundary layer. The temperature reaches its peak at the plate's surface and declines exponentially, approaching zero at a distance from the plate, which aligns with the specified boundary conditions. These figures highlight that the thickness of the thermal boundary layer expands as Bi increases but contracts with higher values of Gr and Pr. Consequently, convective heat transfer at the surface promotes thermal diffusion, whereas an

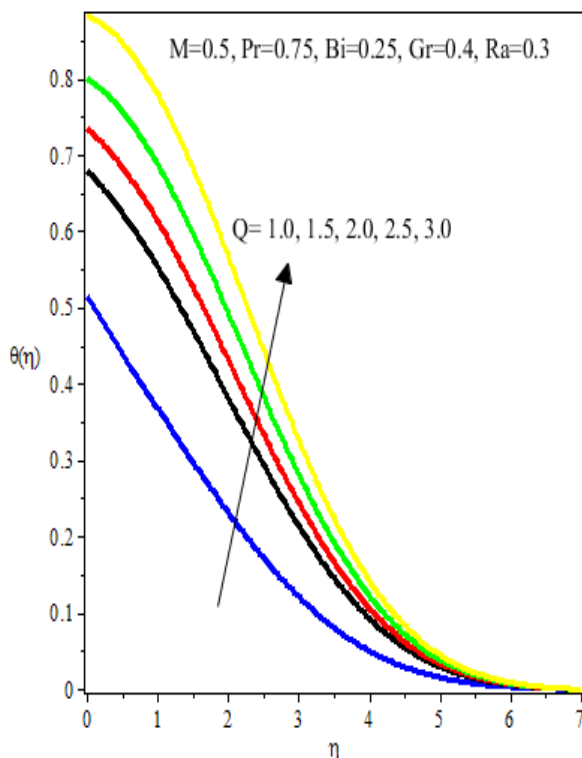


Fig 7: Influence of Q on temperature profile

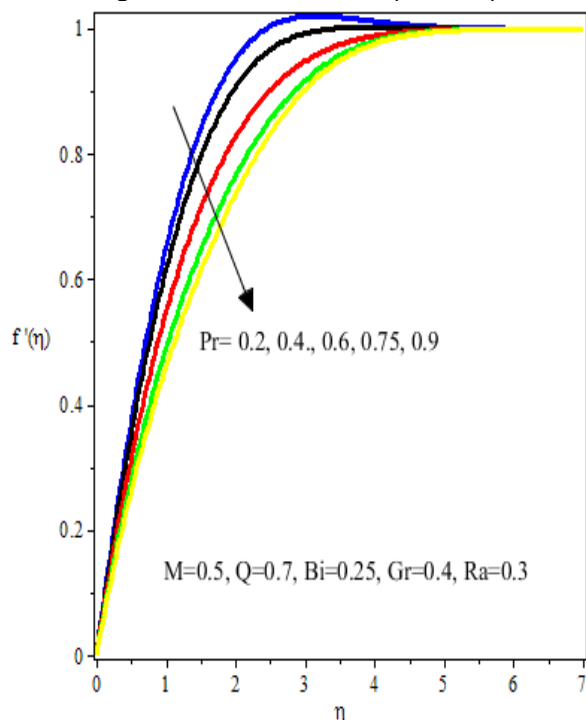


Fig 8: Influence of Pr on temperature profile

increase in the Prandtl number and buoyancy force intensity decelerates the thermal diffusion rate within the boundary layer. Figs. 6 and 7 demonstrate the impact of fluid injection on both velocity and temperature profiles. Notably, introducing heat into the fluid boundary layer results in a rise in both temperature and velocity profiles. In Fig. 8, a higher Prandtl number corresponds to a reduction in the velocity profile.

4. Conclusion

This research examines the boundary layer flow across a vertical plate subjected to a convective surface boundary condition. Similarity solutions are derived for the governing momentum and thermal boundary layer equations under specific assumptions. Numerical methods are employed to solve the similarity equations, and the influence of various parameters on the flow and thermal characteristics is analyzed. The findings indicate that an increase in both the Prandtl number (Pr) and Grashof number (Gr) leads to a combined reduction in the thickness of the thermal boundary layer along the plate. Additionally, the study highlights the significant impact of embedded fluid flow parameters on the thermophysical properties of the fluid. The effects of the Prandtl number and other relevant parameters are further explored to understand their influence on the temperature and concentration profiles of the fluid under different conditions.

Conflict of interest

The authors declare no conflict of interest.

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