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Evaluation of Mixed Convection-Radiation Flow of a Viscous Fluid Restricted to a Vertical Porous Channel: A Comparative Study

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This article examines the steady mixed convection flow in an upstanding porous channel of a hydrodynamic viscous fluid that is electrically conductive due to the thermal radiation effect. The homotopy perturbation method (HPM) is used to generate analytical solutions to the governing differential equations that characterize the velocity and temperature flow properties. For relevant temperature jump and velocity slip situations, the effects of mixed convection parameters, thermal radiation parameters, and magnetic field effects on the velocity field, skin friction coefficient, temperature distribution, and heat transfer rate have been described. This research also analyses and compares the results obtained by Abbas *et al.* (2020) with this present work when HPM was used for the limiting cases. It is interesting to report that an excellent agreement was established, thereby authenticating and validating the accuracy of HPM as a strong tool for obtaining approximate solutions. According to the results of this study, it is possible to effectively control the velocity and temperature gradients by varying the main relevant parameters such as: the mixed convection parameter for a constant pressure gradient, the thermal radiation parameter, the magnetic field effect, the rarefaction parameter, and the wall ambient temperature difference ratio parameter.

Keywords: Mixed convection, Magneto-hydrodynamics (MHD), Thermal radiation, Vertical porous channel, Homotopy perturbation method (HPM).

1. Introduction

The concept of thermal radiation effect, which is the electromagnetic wave radiation that a surface generates because of its heat, is gaining growing attention, especially when a magnetic field is applied, due to its relevance in constructing different advanced energy conversion systems capable of operating at high temperatures (Jamaludin *et al.* 2020). Some other practical applications include nuclear plants, solar technology, spacecraft aerodynamics, and so forth. To this end, several scholars have conducted research on the influence of thermal radiation in a variety of physical settings. Shah *et al*. (2023) elaborated on the influence of heat-dependent thermodynamic fluid properties in MHD Casson flow affected by the coexistence of thermal and chemical reactions. Ojemeri *et al*. (2023) proposed the hydro magnetic natural flow of an electrically conductive Casson fluid due to the thermal radiation impact in an upright porous channel, Using Darcy's model, Gireesha *et al.* (2020) outlined how thermal radiation and free convection affected the flow of a water-based hybrid nanofluid containing nanoparticles through a porous vertical channel, and Goud *et al.* (2023) investigated the analysis of transient MHD flow through a permeable medium across an upright plate in the context of the coexistence of viscous dissipation and thermal radiation effects. The consequences of thermal radiation, a heat source, and an induced magnetic field on the natural convective flow of a couple stress fluid in a flux-isothermal vertical channel have been evaluated by Hasan *et al*. (2020) using the method of an indeterminate coefficient. Using the quasi-linearization technique, Kaladhar *et al.* (2016) highlighted couple stress fluid mixed convection in an upright channel in the coexistence of thermal radiation and Soret components. In the presence of thermal radiation, Bejawada and Nandeppanavar (2023) studied the effects of the MHD heat transfer problem on the micropolar fluid through a vertically permeable moving plate. Parthiban and Prasad (2023) outlined a theoretical investigation of radiative-convection effects on MHD fluid flow in a heated square enclosure having a non-Darcy square cavity in the coexistence of the Hall effect and the heat source or sink. Using a spectral relaxation method, Haroun et al. (2017) searched for the impact of heat radiation on hydro-magnetic mixed convection nanofluid flow across a stretching plate. The effects of combined convective radiation on the stagnation point flow of nanofluid across a stretching or contracting sheet with porosity implications have been highlighted by Pal and Mandal (2015) in the context of viscous dissipation and heat source coexistence.

Engineering, aerodynamics, space research, and medicine could all benefit greatly from the mechanical effects known as suction or injection, which is used to control energy losses near the boundary layer by reducing friction on the surfaces. These applications have piqued our curiosity about the function that suction and injection play in various types of fluids. Due to thermal radiation, Saidulu and Reddy (2023) studied the implications of suction and injection on heat and mass transport in free MHD convection over a stretched permeable plate. Jha *et al*. (2022) employed the Laplace transform method to study how suction or injection affected a natural convective heat flow caused by a point or line heat generation or absorption in an upright channel. As presented by Jha *et al.* (2019), suction and injection have an impact on hydro magnetic free convection flow. From their computational analysis, they concluded that increasing velocity decreases injection parameters. With increasing suction parameters, on the other hand, the shear stress is favored. Jha *et al.* (2018) have analyzed carefully the influence of suction and injection on a time-dependent free convection flow. Suction can be utilized to evacuate reacting substances from chemical reactions, whereas injection can be used to introduce reactants. Falade *et al*. (2017) highlighted the influence of suction and injection on the unsteady oscillatory hydro magnetic flow. It is noteworthy that hot plate injection increases the channel's sheer stress. By periodically heating a vertical surface with heat generation and absorption, Keshtkar et al. (2014) investigated the suction/injection influence exerted by a nanofluid on hydro magnetic mixed convection boundary surface flow. The effects of suction and injection in a steady MHD flow of electrically conductive fluid across an annular porous region in two coaxial cylinders have been discussed by Hamza (2019). The functions of suction and injection on free convection across a constant-heat vertical porous channel in the coexistence of chemical processes and thermal radiation were analytically solved by Usman *et al.* (2022) using an implicit finite difference scheme. Rehman *et al.* (2019) demonstrate the effects of suction and injection on a transient MHD Casson thin film flow with slip and uniform thickness across a stretch film under different flow conditions. The importance of suction/injection, heat transfer, and unsteady hydromagnetic flux over a stretchable rotating plate was discussed by Prasad *et al.* (2020). The impact of multi-slip implications on steady-state MHD fluid flow when Soret and Dufour occur by suction or injection along a non-isothermal stretching surface was reviewed by Reddy *et al*. (2022). Using the Cattaneo-Christov model, Reddy and Lakshminarayana (2022) showed how radiation, chemical reaction, heat generation, suction or injection, and stretching sheets affect the Williamson nanofluid when passing through permeability media.

This research attempts to extend the work of Abbas *et al*. (2020) by incorporating the effects of mixed convective heat transfer with constant pressure on hydro magnetic flow influenced by thermal radiation in a vertical porous channel using HPM. With the help of line graphs, the deviations of different influencing parameters are thoroughly examined. The novelty of this article exists in the sense that it serves as a comparative study where the results obtained by Abbas *et al.* (2020) are compared with the current work when HPM was used to derive the analytical solutions for limiting cases. The comparison demonstrates excellent agreement, thereby validating the accuracy and efficacy of HPM as a reliable procedure for solving both nonlinear and linear differential equations. In many biological and industrial activities, heat and mass transfer take place simultaneously due to an induced buoyancy force. Moreover, under the influence of magnetic fields, these flows have several uses in biological sciences and polymer industries. In addition, these fluids have diverse rheological properties, which increases their relevance in the fields of chemical and biomedical sciences, chemical engineering, and pharmaceuticals, among others. Finally, the findings from this research are essential in establishing the bases for authenticating the accuracy of some other empirical or numerical methods.

2. Structure of the Flow system

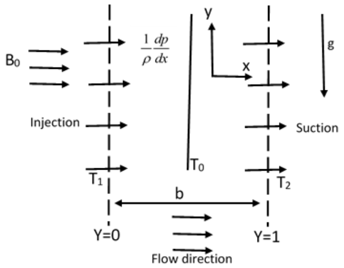

Figure 1: Geometry of the Flow System

Premised on these assumptions, temperature-balanced equations with thermal radiation and the fluid flow due to mixed convection with constant pressure gradient become;

$$\frac{\partial v}{\partial y} = 0 \quad (1)$$

$$v\frac{\partial^2 u}{\partial y^2} - R\frac{\partial u}{\partial y} + g\beta(T - T_0) - \frac{\sigma B_0^2 u}{\rho} = -\frac{\partial P}{\partial x} \quad (2)$$

$$R\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \quad (3)$$

Applying the Rosseland approximation, the radiative heat flux q_r is defined as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (4)$$

The Stefan-Boltzmann constant and Rosseland mean absorption coefficient are indicated by the symbols σ^* and k^*, respectively. Assume that T^4 can be written as a linear function of temperature in a Taylor series around T_0, ignoring higher terms, since the temperature difference inside the flow is sufficiently negligible, we have

$$T^4 \cong 4T_0^3 T - 3T_0^4 \quad (5)$$

From equations (4) and (5), equation (3) becomes

$$\frac{k}{\rho c_p}\left(1 + \frac{16\sigma^* T^3}{3kk^*}\right)\frac{\partial^2 T}{\partial y^2} - R\frac{\partial T}{\partial y} = 0 \quad (6)$$

where $D = 16\sigma^* T^3/3k k^*$ is considered a radiation parameter, hence from the equation above, we obtain

$$\frac{k}{\rho c_p}(1 + D)\frac{\partial^2 T}{\partial y^2} - R\frac{\partial T}{\partial y} = 0 \quad (7)$$

Now, we introduce the dimensionless parameters as

$$y = \frac{y\prime}{b}, u = \frac{u\prime}{U_0}, \theta = \frac{T - T_0}{T_1 - T_0}, Re = \frac{U_0 x}{v}, \ Gr = \frac{g\beta(T_w - T_0)h^3}{v^2}, \gamma = -\frac{dP}{dx}, \quad (8)$$

Where $U_0 = \rho g\beta(T - T_0)b^2/\mu$ is dimensionless velocity.

According to Abbas et al. (2020), by integrating equation (8) into equations (2) and (3):

$$\frac{d^2u}{dy^2} - R\frac{du}{dy} - M^2 u + \frac{Gr}{Re}\theta = \gamma \quad (9)$$

$$(1 + D)\ \frac{d^2\theta}{dy^2} - RPr\frac{d\theta}{dy} = 0 \quad (10)$$

The following boundary conditions apply in the dimensionless area:

$$\left.\begin{aligned} \theta(0) &= \xi + \alpha_1\frac{d\theta}{dy},\ U(0) = \alpha_2\frac{dU}{dy} \\ \theta(1) &= 1 - \alpha_1\frac{d\theta}{dy},\ U(1) = -\alpha_2\frac{dU}{dy} \end{aligned}\right\} \quad (11)$$

where R is the suction/injection parameter, M is the magnetic field intensity, $\frac{Gr}{Re} = Gre$ is the mixed convection parameter, γ is the pressure gradient, D is the thermal radiation effect, Pr is the Prandlt number, ξ is the wall ambient temperature difference ratio, $\alpha_1 = \beta_v KnIn$ is the temperature jump coefficient and $\alpha_2 = \beta_v Kn$ is the velocity slip condition.

3. Solution of the Problem

3.1 Basic idea of Homotopy perturbation method (HPM)

To demonstrate the fundamental concepts of HPM, look at the nonlinear differential equation below:

$$A(u) - f(r) = 0, \quad (12)$$

with the boundary conditions

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \qquad r \in \Gamma,$$

where A is a general differential operator, B is a boundary operator $f(r)$, denotes a known analytical function, and Γ is the domain boundary Ω. In general, operator A can be divided into two parts which are L and N, where L refers to the linear part and N to the nonlinear part. As a result, (12) can be expressed as follows:

$$L(u) + N(u) - f(r) = 0, \qquad r \in \Omega$$

With the homotopy technique, we construct a homotopy as follows:
$v(r,p):\Omega\times[0,1]\to R$ which satisfies:

$$H(v,p)=(1-p)[L(v)-L(u_0)]+p[A(v)-f(r)]=0 \quad (15)$$

where u_0 is an initial approximation of equation (12), which satisfies the boundary conditions, and $p \in [0, 1]$ is an embedding parameter. We can see from eqn (15) that we have

$$H(v,0)=L(v)-L(u_0)=0 \quad (16)$$

$$H(v,1)=A(v)-f(r)=0 \quad (17)$$

Only the process of changing $v(r,p)$ from the initial approximation solution $u_0(r)$ to the final solution u(r) causes p to change from zero to unity. This is referred to as deformation in topology, while homotopy refers to the relationships $L(v)-L(u_0)$ and $A(v)-f(r)$. As per HPM, the first "small parameter" we can employ is the embedding parameter p. Assume that equation (15) has a solution that can be expressed as a power series in p:

$$v=v_0+pv_1+p^2v_2+p^3v_3+\ldots \quad (18)$$

setting $p = 1$ gives the approximate solution of eqn (1) as

$$u=\lim_{n\to\infty}v=v_0+v_1+v_2+v_3+\ldots \quad (19)$$

$$u=v_0+v_1+v_2+v_3+\ldots \quad (20)$$

The HPM, which does not have the restrictions of conventional perturbation methods, combines the perturbation method and the homotopy method. For the few early terms, the series equation (19) is convergent in the majority of situations. However, He (1999, 2000, 2003), and Ayati and Biazar (2015), to name a few, have shown that the convergence rate relies on the nonlinear operator A(v).

Constructing a convex homotopy on equations (9) and (10) we have:

$$H(U,p)=(1-p)\frac{d^2U}{dy^2}-p\left[R\frac{du}{dy}+M^2U-\frac{Gr}{\mathrm{Re}}\theta+\gamma\right]=0 \quad (21)$$

$$H(\theta,p)=(1-p)\frac{d^2\theta}{dy^2}-p\left[R\frac{d^2\theta}{dy^2}+K_1\frac{d\theta}{dy}\right]=0 \quad (22)$$

Assuming that the solutions of U and θ exist in an infinite series, the following form results:

$$\left.\begin{aligned}\theta(y)&=\theta_0+p\theta_1+p^2\theta_2+\ldots\\ U(y)&=u_0+pu_1+p^2u_2+\ldots\end{aligned}\right\} \quad (23)$$

Hence, the approximate solutions for the temperature and velocity subject to the transformed and corresponding boundary conditions are derived as:

$$\theta_0(y)=C_1y+C_2 \quad (24)$$

$$\theta_1(y)=C_3y+C_4+k_1C_1\frac{y^2}{2} \quad (25)$$

$$\theta_2(y)=C_5y+C_6+k_1C_3\frac{y^2}{2}+k_1^2C_1\frac{y^3}{6} \quad (26)$$

$$u_0(y)=Z_1y+Z_2 \quad (27)$$

$$u_1(y)=Z_3y+Z_4-Gre\left[\left[C_1\frac{y^3}{6}+C_2\frac{y^2}{2}\right]\right]+\gamma\frac{y^2}{2} \quad (28)$$

$$u_2(y)=Z_5y+Z_6+R\left[Z_3\frac{y^2}{2}-Gre\left(C_1\frac{y^4}{24}+C_2\frac{y^3}{6}\right)+\gamma\frac{y^3}{6}\right]+M^2\left[Z_3\frac{y^3}{6}+Z_4\frac{y^2}{2}-Gre\left(C_1\frac{y^5}{220}+C_2\frac{y^4}{24}\right)+\gamma\frac{y^4}{24}\right]-Gre\left[\left[C_3\frac{y^3}{6}+C_4\frac{y^2}{2}+k_1C_1\frac{y^4}{24}\right]\right] \quad (29)$$

Therefore, setting p = 1 gives the approximate solutions of θ and U as

$$\theta(y)=\theta_0+\theta_1+\theta_2+\ldots \quad (30)$$

$$U(y)=u_0+u_1+u_2+\ldots \quad (31)$$

The rate of heat transfers and sheer stress at both plates are calculated as

$$\left.\frac{d\theta}{dy}\right]_{y=0,1} \quad (32)$$

and

$$\left.\frac{du}{dy}\right]_{y=0,1} \quad (33)$$

4. Discussion of Results

The implications of steady mixed convection-radiation flow of a viscous fluid in the coexistence of thermal radiation and suction/injection effects in a vertical porous channel are investigated. The flow characteristics of temperature, momentum, rate of heat transfers, and frictional force coefficient under the influence of diverse and major controlling factors such as: magnetic field effect, thermal radiation parameter, mixed convection parameter, and wall ambient temperature difference ratio has been thoroughly discussed, and the graphically presented computed analysis results are described in Figures 2-8. The current parametric computations have been carried out over a reasonable range of 0 $\leq (\beta_v Kn) \leq$1, -1 ≤ ξ ≤ 1, 20 ≤ Gre ≤ 100, 5 ≤ D ≤ 1, 0.5 ≤ M ≤ 2, -0.5 ≤ R≤ 0.5, 0.71 ≤ Pr ≤7 and 0≤In≤10. While (In) indicates a feature of the fluid-wall interaction, the product represents a measure of deviation from the continuum regime. The default values selected for this present analysis are Gre=20. D= 2, M = s = 0.5, γ=1 and Pr =0.71. The analytical results obtained by Abbas *et al*. (2020) was compared with this current investigation when HPM was used for limiting cases as indicated in Table 1. The comparison confirms an excellent agreement.

The functions of thermal radiation on the temperature and velocity gradients for each of the three cases of wall-ambient temperature difference ratios (ξ = -1: one wall is cooling and the other wall is heating; ξ =0: one wall is not heating and the other wall is heating, ξ =1: the case were both walls are heated) are shown in Figures 2 and 3. From these figures, it is clear that as the level of thermal radiation increases, so does the temperature and flow of the fluid. It is however important to report that at ξ =1, thermal radiation has insignificant effect on the heat or velocity profiles.

Figure 4 depicts the effect of Gre on the fluid motion. It can be seen in this plot that as the mixed convection parameter increases, the velocity increases. For different ascending values of ξ.

The consequence of MHD on the velocity gradient is portrayed in Figure 5. The pattern shows a decrease in fluid flow with increasing magnetic field strength. This is due to the Lorentz force, which is present when a magnetic field is applied to an electrically conductive fluid and a resistance force is produced. This force causes the fluid flow to slow down as it approaches the plate. As a result, all other forces, including the Lorentz force, vanish when the fluid comes to rest.

The variation of thermal radiation factor versus $\beta_v Kn$ parameter for heat flow rates is plotted in Figure 6 for ascending values of ξ. It is evident that the same increasing tendencies were observed on the both microchannel walls.

The effect of thermal radiation on wall shear stress is described with reference to Figure 7. At wall temperature $y=0$, skin friction increases with increasing thermal radiation values, but at $y=1$ the trend is reversed, as shown in Figure 7b.

The behavior of Gre at both micro-channel surfaces for the frictional force is portrayed in Figures 8a and b. At the wall (y=0), there is a considerable increase in skin friction, as shown in Fig. 8a, however at y=1, the opposite phenomena occur as described in Figure 8b.

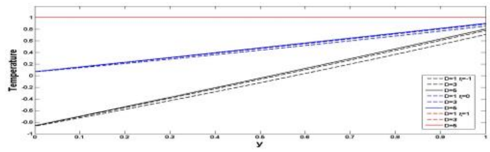

Figure 2: Thermal radiation action on the temperature profile

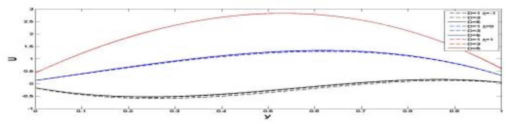

Figure 3: Thermal radiation action on the velocity profile

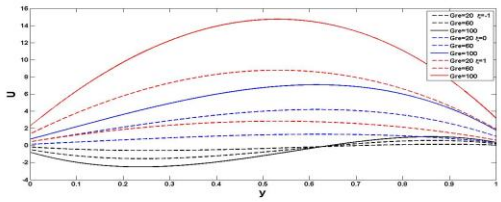

Figure 4: Mixed convection action on the velocity profile

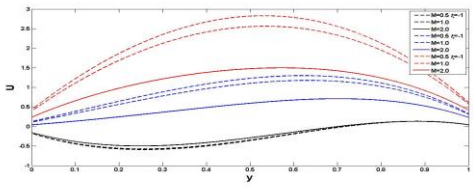

Figure 5: Magnetic field action on the velocity profile

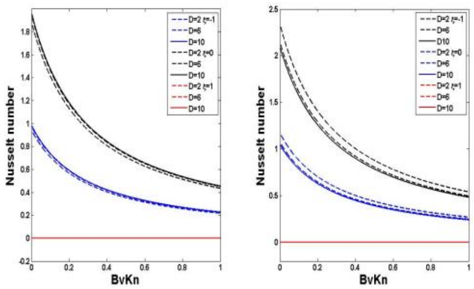

Figure 6: Thermal radiation action on Nusselt number

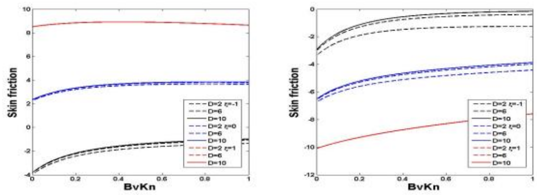

Figure 7: Thermal radiation action on Skin friction

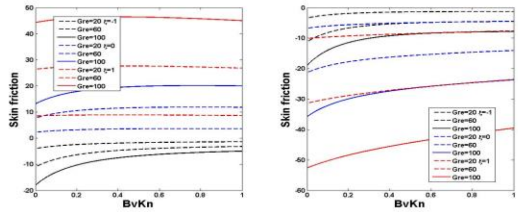

Figure 8: Mixed convection action on Skin friction

5. Validation of Results

The numerical computation of the comparison between the work of Abbas *et al*. (2020) with the present work has been computed. The comparison portrays an excellent agreement.

Table 1: Numerical comparison of the current study on the velocity profile with the work of Abbas *et al.* (2020), for different $\xi = -1, 0, 1$ at $\beta_v Kn$=0.05, Pr=0.71, In=1.667, M=s=1, R=0.5, setting Gre=1 and γ=0.

Y	Abbas *et al.* (2020) U(Y)			Current study U(Y)		
	$\xi = -1$	$\xi = 0$	$\xi = 1$	$\xi = -1$	$\xi = 0$	$\xi = 1$
0.1	0.6012	0.2741	-0.0530	0.6012	0.2741	-0.0530
0.2	0.9158	0.4154	-0.0850	0.9158	0.4154	-0.0850
0.3	1.1560	0.5216	-0.1127	1.1560	0.5216	-0.1127
0.4	1.3194	0.5922	-0.1350	1.3194	0.5922	-0.1350
0.5	1.4016	0.6257	-0.1502	1.4016	0.6257	-0.1502

6. Conclusion

The effect of steady-state MHD mixed convection flow of an incompressible viscous fluid in the coexistence of radiation and wall porosity effects has been investigated. One of the key achievements of this research is obtaining the same results as Abbas *et al* (2020), when the homotopy perturbation method was used to solve the current problem for the limiting cases, thereby qualifying this research as a comparative study. It can also be deduced from this article that HPM can be viewed as a reliable and trustable tool for solving both linear and nonlinear differential equations The closed solutions have been derived for the thermal, momentum, Nusselt number, and drag force coefficients with the aid of different plots so as to showcase the fluctuations pattern for the main controlling parameters embedded in the flow configuration. The major outcomes of this study are highlighted as follows:

i. The work of Abbas *et al*. (2020) was successfully recovered when the influence of mixed convection and pressure gradient are neglected.

ii. Mixed convection parameter favors the fluid motion for ascending values of wall ambient temperature difference ratio

iii. The actions of thermal radiation and rarefaction factors are seen to encourage the fluid temperature and fluid acceleration respectively for growing values of wall ambient temperature difference ratio.

iv. Increasing the magnetic number impedes the fluid movement due to the Lorentz force action

v. Uplifting the mixed convection parameter, the frictional force affected by the rarefaction effect at y=0 escalates, while a counter attribute happens at y=1.

vi. The employed method (HPM) demonstrates an excellent potential in respect to accuracy and convergence for simulating flow.

Conflict of interest

The authors declare no conflict of interest.

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