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Permutation graphs with co-inversion on Γ_1-non-deranged permutations

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In this paper, we define permutation graphs on Γ_1-non-deranged permutations using the set of co-inversion as edge set, and the values of permutation as the set of vertices. From the graphs, we observed that diameter and radius of the graph of any ω_1 is one, the graph of any $\omega_{p-1} \in G_p^{\Gamma_1}$ is simple, the graph of ω_1 is completed and other properties of the graphs were also observed.

Keywords: Permutation graphs, Γ_1-non-deranged permutations, co-Inversion, completed graphs and simple graphs.

1. Introduction

A co-inversion of the permutation f is a pair (i, j) such that $i < j$ and $f(i) < f(j)$ which is denoted as $Coinv(f)$ and the number of co-inversion f denoted by $coinv(f) = |Coinv(f)|$. The concept of derangements in permutation groups (that is permutations without a fix element) has proportion in the underlying symmetric group S_n [1] used the concept to develop a scheme for prime numbers $P \geq 5$and $\Omega \subseteq N$ which generate the cycles of permutations (derangements) using $\omega_i = ((1)(1+i)_{mp}(1+2i)_{mp}...(1+(p-1)i)_{mp})$ to determine the arrangements. It is difficultfor a set of derangements to be a permutation group because of the absence of the natural identity element (a non- derangement). The construction of the generated set of permutations from the work of [1] as a permutation group was studied by [2]. They achieved this by embedding an identity element (although not a permutation) into the generated set of permutation (strictly derangements) with the natural permutation composition as the binary operation (the group was denoted as G_P Some of the algebraic properties of the structure were investigated and some fascinating results were obtained. A study on this permutation group G_P by [3] is the extension of the permutation group G_P (of prime number of entries greater than or equal to 5) by modular arithmetic and concatenation map in which another algebraic structure G_P' was obtained and its group theoretic properties were studied, which shows that the group is Abelian. [4] modified the scheme of [1] to two-line notation and the scheme generated a set of permutations with a fix at 1 (which generated the natural identity). This obtained set of permutations form permutation group called the Γ_1 non-deranged permutation group and is denoted as $G_P^{\Gamma_1}$. [5] studied the Fuzzy ideal of function f Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ and established that it is one side fuzzy (only right fuzzy but not left) also the α-level cut of f coincides with $G_P^{\Gamma_1}$ if $\alpha = \frac{1}{p}$. [6] considered ascent on Γ_1-non deranged permutation group $G_P^{\Gamma_1}$ in which recursion formula for generating Ascent number, Ascent bottom and Ascent top was develop and also observe that $Asc(\omega_i)$ union $Asc(\omega_{p-1})$ is equal to $Asc(\omega_1)$. [7] provided very useful theoretical properties of Γ_1-non deranged permutation s in relation to exceedance and shown that the exceedance set of all ω_i in $G_P^{\Gamma_1}$ such that $\omega_i \neq e$ is $\frac{1}{2}(p-1)$.

[8] established that the intersection of descent set of all Γ_1-non derangement is empty, also observed that the descent number is strictly less than one [9] established that inversion number and major index are not equidistributed in Γ_1-non deranged permutations and also established that the difference between sum of the major index and sum of the inversion number is equal to sum of descent number in Γ_1-non deranged permutations. [10] studied standard representation of Γ_1-non deranged permutations and also identified relation to ascent block by partitioning the permutation set in which a recursion formula for generating maximum number of block and minimum number of block were develop and it is also observed $ar(\omega_i)$ that is equidistributed with $asc(\omega_i)$ for any arbitrary permutation group. [11] recognized that in Γ_1-non deranged permutations, the radius of a graph of any ω_1 is zero, the graph of any $\omega_i \in G_p^{\Gamma_1}$ is null, and by restricting 1, the graph of ω_{p-1} is complete. [12] established that the Right embracing number of Γ_1-non deranged permutations of ω_i $\mathrm{Re}\,s(\omega_i)$ is equidistributed with the Left embracing $Les(\omega_i)$ and then $\mathrm{Re}\,s(\omega_i)$ is equidistributed with $\mathrm{Re}\,s(\omega_{p-i})$ and also observed that the height of weighted motzkin path of ω_i is the same as the height of weighted motzkin path of $\omega_{p-des(\omega_i)}$ [13] investigated some algebraic theoretic properties of fuzzy set on $G_p^{'}$ using constructed membership function of fuzzy set on $G_p^{'}$ and established the result for algebraic operators of fuzzy set on $G_p^{'}$ which are algebraic sum, algebraic product, bounded sum and bounded difference and also constructed a relationship between the operators and fuzzy set on $G_p^{'}$. [14] studied partition block coordinate statistics on Γ_1-non-deranged permutations and observed that left opener bigger block $lobTC(\omega_i)$ is equidistributed with right opener bigger block $robTC(\omega_i)$. More recently [15] established that the admissible inversion descent $aid(\omega_{p-1})$ is equi-distributed with descent number $des(\omega_{p-1})$ and also showed that the admissible inversion set $Ai(\omega_i)$ and admissible inversion set $Ai(\omega_{p-i})$ are disjoint. Hence, we will in this paper show that the diameter and radius of the graph of any ω_1 is one, the graph of ω_1 is complete and also showed that the graph of any $\omega_{p-1} \in G_p^{\Gamma_1}$ is simple.

2. Materials and Methods

In this section before we outline the main result results in this research paper, we attempt to define some basic concept that will help in further understanding of this work

Definition 2.1 [15]

Let Γ be a non-empty set of prime cardinality $p \geq 5$ such that $\Gamma \subset N$ A bijection ω on Γ of the form

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & . & . & . & p \\ 1 & (1+i)_{mop} & (1+2i)_{mop} & . & . & . & (1+(p-1)i)_{mop} \end{pmatrix}$$

is called a Γ_1-non-deranged permutation. We denoted G_p to be the set of all Γ_1-non-deranged permutations.

$G_7 = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6\}$ is the set of all Γ_1-non-deranged permutations where $p = 7$

By definition 2.1, G_7 is generated as follows:

$$\omega_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix}$$

$$\omega_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 5 & 7 & 2 & 4 & 6 \end{pmatrix}$$

$$\omega_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 7 & 3 & 6 & 2 & 5 \end{pmatrix}$$

$$\omega_4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 5 & 2 & 6 & 3 & 7 & 4 \end{pmatrix}$$

$$\omega_5 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 6 & 4 & 2 & 7 & 5 & 3 \end{pmatrix}$$

$$\omega_6 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 7 & 6 & 5 & 4 & 3 & 2 \end{pmatrix}$$

Definition 2.2 [9]

The pair G_p and the natural permutation com position forms a group which is denoted as $G_P{}^{\Gamma_1}$.This is a special permutation group which fixes the first element of Γ.

Definition 2.3 [4]

A co- inversion of permutation $f = \begin{pmatrix} 1 & 2 & 3 & . & . & . & n \\ f(1) & f(2) & f(3) & . & . & . & f(n) \end{pmatrix}$ is a pair (i, j) such that $i < j$ and $f(i) < f(j)$.The co-inversion set of f, denoted as $Coinv(f)$, is given by

$$Coinv(f) = \{(i, j) : 1 \le i < j \le n \text{ and } f(i) < f(j)\}$$

, the co-inversion number of f, denoted by $coinv(f) = |Coinv(f)|$.

Example 2.1

For ω_4 in $G_5^{\Gamma_1}$

$$\omega_4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 4 & 3 & 2 \end{pmatrix}$$

$$Coinv(\omega_4) = \{(1,2),(1,3),(1,4),(1,5)\}$$

$$coinv(\omega_4) = 4$$

Definition 2.4

Permutation graph is a graph whose vertices represent the element of a permutation and whose edges represent the pairs of element that are reversed by the permutation.

3. Results and Discussion

In this section, we discuss the details of the investigations and results obtain.

Proposition 3.1

Let $\omega_{p-1} \in G_P{}^{\Gamma_1}$.Then the graph $G\omega_{p-1}$ is simple

Proof:

For any $G_P{}^{\Gamma_1}$,

$\omega_{p-1} = \begin{pmatrix} 1 & 2 & 3 & . & . & . & p \\ 1 & p & p-1 & . & . & . & 2 \end{pmatrix}$. Theref ore $Coinv(\omega_{p-1}) = \{(1,2),...,(1,p-1)(1,p)\}$ by using the set of co-inversion as the edge set, and value of permutation as the set of vertices, we observed that no pair is repeated an no pair has some coordinates, so the graph has neither loop nor multiple edges. Hence its simple

Proposition 3.2

For any $\omega_1 \in G_P{}^{\Gamma_1}$.The graph $G\omega_1$ is complete

Proof:

For any $G_P{}^{\Gamma_1}$,

$$\omega_1 = \begin{pmatrix} 1 & 2 & . & . & . & p-1 & p \\ 1 & 2 & . & . & . & p-1 & p \end{pmatrix}.$$

by definition of co-inversion, it will consist of all possible pairs of the letters of ω_1 Hence all vertex is adjacent to each other this complete the proof.

Corollary 3.3

For any $\omega_1 \in G_P{}^{\Gamma_1}$. The graph $G\omega_1$ is regular

Proof.

It is obvious that every complete graph is regular, thus the result shows by proposition 3.2

Proposition 3.4

For any $\omega_1 \in G_P{}^{\Gamma_1}$. Then the graph of ω_1 has one component

Proof:

The vertex (1) is adjacent to every vertex hence no vertex or vertices are isolated, this it has one component. Hence the proof.

Proposition 3.5

The graph $G\omega_{p-1}$ is a tree in Γ_1-non-deranged permutations.

Proof:

For any $\omega_1 \in G_P{}^{\Gamma_1}$ then, ω_{p-1} is of the form $\omega_{p-1} = 1\ p\ p-1 \ldots 2$ therefore by computing the co-inversion, 1 is adjacent to all letters while the relaxing 1, the other letters are in decreasing sequence, so no vertex is adjacent to another. Hence vertex 1 is the root vertex. Hence $G\omega_{p-1}$ has no cycle. Therefore, it is a cyclic.

Proposition 3.6

For any $\omega_1 \in G_P^{\Gamma_1}$. Then the $diam(G\omega_1) = rad(G\omega_1) = 1.$

Proof:

The result follows since the graph of $(G\omega_1)$ is always complete

Proposition 3.7

Let $\omega_1 \in G_P^{\Gamma_1}$. Then, the maximum degree $\Delta(G\omega_1) = P - 1.$

Proof:

We have that for any $\Delta(G\omega_1)$ is adjacent to all other vertices. So vertex (1) is adjacent to $p-1$ vertices, therefore $\deg(1) = p-1$ which is the maximum vertex with maximum degree. Hence $\Delta(G\omega_1) = p-1$

Proposition 3.8

For any $\omega_{p-1} \in G_P^{\Gamma_1}$. Then the

$$E(G\omega_{p-1}) = \bigcup_{k=1}^{p-1}(1, k+1)$$

Proof:

For any $G_P^{\Gamma_1}$, $\omega_{p-1} = 1\,p\,(p-1)\ldots 2$ t so we can have co-inversion at letter (1) only, which is the least and it is at the left most hand side. Therefore, the co-inversion contains only the pairs of (1) with each letter that is $Coinv(\omega_{p-1}) = \bigcup_{k=1}^{p-1}(1, k+1)$ and since $Coinv(\omega_{p-1}) = E(G\omega_{p-1})$ The result follows.

Proposition 3.9

Let $\omega_i \in G_P^{\Gamma_1}$. Then the

$$E(G\omega_i) \cup E(G\omega_{p-i}) = E(G\omega_{p-1})$$

Proof:

Suppose $\omega_i = a_1, a_2, a_3, \ldots, a_{p-1}, a_p$ then $\omega_{p-i} = a_1, a_p, a_{p-1}, \ldots, a_2$ so by restricting a_1, since it's the least and the first position, ω_i is the reverse of ω_{p-i} .Hence the $Coinv(\omega_i)\backslash 1 \cap Coinv(\omega_{p-i})\backslash 1 = \varnothing$, hence the letters which pairs contains 1.

4. Conclusion

This paper has extended permutation graph with inversion of [11] to permutation graph with co-inversion in which we have shown that the graph of any $\omega_{p-1} \in G_p^{\Gamma_1}$ is simple and also shown that the diameter and radius of the graph of any ω_1 in Γ_1- non-deranged permutations is one and complete.

Conflict of Interest

The author declares that there is no conflict of interest.

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