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On the efficiency of modified regression-type mean imputation scheme under two-phase sampling

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Human-based surveys such as medical and social science surveys are often characterized by non-response or missing observations. In this study, a new class of regression-type mean imputation method that uses $\bar{x}_n$ as an estimate of $\bar{X}$ was suggested. Using partial derivative approach, the MSEs of the class of estimators presented were derived up to first order approximation under two cases. Case I: when the secondary sample S_2 of size $n\ (n < n_1)$ is a subset of preliminary sample $S_1\ [S_2 \subset S_1]$, and Case II: is when secondary sample S_2 is a subset of universal set Ω_N. Conditions for which the new estimator was more efficient than the other estimators studied were derived. The results of numerical examples through simulations revealed that the suggested class of estimators is more efficient.

Keywords: Imputation method, Secondary sample, Preliminary sample, missing observations.

1. Introduction

Several studies in the field of sampling survey have developed numerous estimators for estimating population parameters like population mean, population variance, standard deviation etc. under the assumption that complete information about sampling units is available. Authors like [1], [2], [3], [4], [5], [6], [7] have worked extensively in that direction. However, authors like [8], Sisodia et al. [9], [10], [11], [12], [13], [14], [15], [16], and [17] utilized coefficient of variation of auxiliary variable in the estimators' formulation and obtained highly efficient estimators. The estimators in the aforementioned literatures assumed that information on sampling units drawn from the population is completely available, this assumption is often violated due to non-response due to incomplete information or inaccessibility to respondents or refusal to answer questions. In such situations, responses of non-respondents after often imputed or estimated using imputation techniques.

Imputation is the process of replacing missing data with substituted values. When substituting for a data point, it is known as "unit imputation"; when substituting for a component of a data point, it is known as "item imputation" ([18]). There are three main problems that missing data causes. It can introduce a substantial amount of bias, make the handling and analysis of the data more arduous, and create reductions in efficiency. Missing data due to non-response can create problems for analyzing data and imputation is seen as a way to avoid pitfalls involved with likewise of cases that have missing value. Imputation preserves all cases by replacing missing data with an estimated value based on other available information. Once all missing values have been imputed, the data set can then be analysed using standard techniques for complete data ([19]). There have been many theories embraced by scientists to account for missing data but the majority of them introduce bias.

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Survey such as in medical and social science etc. conducted by human are often characterized by non-response. [20] first discussed the issue of non-response and imputation methods to deal with non-response issues were suggested by several scholars like [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], Prasad [33], [34], [35], [36], [37], [38], are some of the most recent imputation methods. However, all the estimators of the schemes proposed by aforementioned authors are functions of population mean of auxiliary variable ($\bar{X}$) and if $\bar{X}$ is unknown, the schemes can not be applied to real life situations. This study, therefore, utilize the concept of two-phase sampling in which a large sample of size $n\ (n<N)$ is taking to estimate $\bar{X}$ thereby addressing the problem of complete information about the auxiliary variable.

1.1 Notations

The following notations have been used.
Y: Study variable. X: Auxiliary variable.
r: number of respondents
n: Size of the sample selected from the entire population.
$\bar{x}_n$: Sample mean for X based on the sample of size n.
$\bar{x}_r$: Sample mean for X based on the respondents (r).
$\bar{y}_r$: Sample mean for Y based on the respondents (r).
$\bar{Y} = N^{-1}\sum_{i=1}^{N} y_i$: Population mean of Y
$\bar{X} = N^{-1}\sum_{i=1}^{N} x_i$: Population mean of X
$S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2$: Population mean squares of X.
$S_Y^2 = (N-1)^{-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^2$: Population mean squares of Y.
$S_{YX} = \rho S_Y S_X$. $\psi_{r,N} = r^{-1}-N^{-1}$,
$\psi_{r,n} = r^{-1}-n^{-1}$ $\psi_{r,n} = r^{-1}-n^{-1}$
$C_Y = S_Y/\bar{Y}$, $C_X = S_X/\bar{X}$: $R=\bar{Y}/\bar{X}$:

$$\rho = \frac{\sum_{i=1}^{N}\left(X_i-\bar{X}\right)\left(Y_i-\bar{Y}\right)}{\sqrt{\sum_{i=1}^{N}\left(X_i-\bar{X}\right)^2\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)}} :$$

1.2 Some Existing Imputation Schemes and their Estimators

Let Φ denotes the set of r units' response and Φ^c denotes the set of $n-r$ units' non-response or missing out of sample space Ω_n of n units sampled without replacement from the N units' population Ω_N such that $\Phi\cup\Phi^c=\Omega_n$ and $\Phi\cap\Phi^c=\{\ \}$. For each $i\in\Phi$, the value of y_i is observed. However, for unit $i\in\Phi^c$, y_i is missing but can be computed using different methods of imputation.
Under mean method of imputation, values found missing due to non-response are to be replaced by the mean of the rest of observed values ([39]). The study variable thereafter, takes the form given as,

$$y_{.i} = \begin{cases} y_i & \mathrm{i}\in\Phi \\ \bar{y}_r & \mathrm{i}\in\Phi^c \end{cases} \qquad (1)$$

Under (2.1), sample mean denoted by $\hat{\mu}_0$ can be derived as

$$\hat{\mu}_0 = r^{-1}\sum_{i\in R} y_r \qquad (2)$$

The bias and variance of $\hat{\mu}_0$ is given in (2.3) and (2.4)

$$Bias(\hat{\mu}_0)=0 \qquad (3)$$

$$Var(\hat{\mu}_0)=MSE(\hat{\mu}_0)=\psi_{r,N}S_Y^2 \qquad (4)$$

The sample mean $\bar{y}_r$ in the imputation scheme defined in (1) is sensitive to extreme values or outliers in data, which in turn depreciate the efficient of the scheme ([18]).
[40] proposed ratio imputation method as given (5)

$$y_{.i} = \begin{cases} y_i & \mathrm{i}\in\Phi \\ \hat{\beta}\mathrm{x_i} & \mathrm{i}\in\Phi^c \end{cases} \qquad (5)$$

where $\hat{\beta}=\sum_{i=1}^{r} y_i / \sum_{i=1}^{r} x_i = \bar{y}_r / \bar{x}_r$

Under (5), estimator of population mean denoted by $\hat{\mu}_1$, as in (6)

$$\hat{\mu}_1 = \hat{\mu}_0\bar{x}_n\bar{x}_r^{-1} \qquad (6)$$

The Bias and MSE of $\hat{\mu}_1$ up $O\left(n^{-1}\right)$ is given as:

$$Bias(\hat{\mu}_1)=\bar{Y}\psi_{r,n}\left(C_x^2-\rho C_x C_y\right) \qquad (7)$$

$$MSE(\hat{\mu}_1)=MSE\left(\hat{\mu}_0\right)+\psi_{r,n}\left(S_y^2+R^2S_X^2-2RS_{YX}\right) \qquad (8)$$

[21] suggested compromised imputation scheme given by (9).

$$y_{.i}=\begin{cases}\lambda\dfrac{n}{r}y_i+(1-\lambda)\hat{\beta}x_i & \mathrm{i}\in\Phi\\ (1-\lambda)\hat{\beta}x_i & \mathrm{i}\in\Phi^c\end{cases} \tag{9}$$

Under (9), estimator of population mean denoted by $\hat{\mu}_2$ can be derived as

$$\hat{\mu}_2=\hat{\mu}_0\left(\lambda+(1-\lambda)\bar{x}_n\bar{x}_r^{-1}\right) \tag{10}$$

$$Bias(\hat{\mu}_0)=\bar{Y}(1-\lambda)\psi_{r,n}\left(C_x^2-\rho C_xC_y\right) \tag{11}$$

$$MSE(\hat{\mu}_2)=MSE(\hat{\mu}_0)-\psi_{r,n}\beta_{rg}\rho_{YX}S_XS_Y \tag{12}$$

where $\lambda=1-\rho\dfrac{C_y}{C_x}$

[23] proposed imputation scheme for population mean estimators which is applicable when the study and auxiliary variables are either positively or negatively correlated, using power transformation.

$$y_{.i}=\begin{cases}y_i, & i\in\Phi\\ \dfrac{1}{n-r}\left[n\bar{y}_r\left(\dfrac{\bar{X}}{\bar{x}}\right)^{\beta}-r\bar{y}_r\right], & i\in\Phi^C\end{cases} \tag{13}$$

Under (13), the resultant estimator of the population mean $\bar{Y}$ given as

$$\hat{\mu}_3=\bar{y}_t\left(\frac{\bar{X}}{\bar{x}_r}\right)^{\beta} \tag{14}$$

The performance of $\hat{\mu}_3$ attained optimality when $\beta=\rho C_y/C_x$

The bias, mean square error and minimum mean square error of $\hat{\mu}_3$ respectively are given in (15) and (16)

$$B(\hat{\mu}_3)=\bar{Y}\psi_{n,N}\left(\frac{\beta(\beta+1)C_X^2}{2}-\beta\rho C_YC_X\right) \tag{15}$$

where $\beta=\rho\dfrac{C_y}{C_x}$

$$MSE(\hat{\mu}_3)_{\min}=\left[\psi_{r,N}(1-\rho^2)\right]S_Y^2 \tag{16}$$

[29] proposed Exponential-Type Compromised Imputation scheme to minimize the effect of distance between $\bar{X}$ and $\bar{x}_r$ on the efficiency of [23] as

$$y_{.i}=\begin{cases}v\dfrac{n}{r}y_i(1-v)\bar{y}_r\exp\left(\dfrac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r}\right) & i\in\Phi\\ (1-v)\bar{y}_r\exp\left(\dfrac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r}\right) & i\in\Phi^C\end{cases} \tag{17}$$

Under (17) the point estimator of population mean $\bar{Y}$ under the proposed method of imputation is:

$$\hat{\mu}_4=v\bar{y}_r+(1-v)\bar{y}_r\exp\left(\left(\bar{X}-\bar{x}_r\right)\left(\bar{X}+\bar{x}_r\right)^{-1}\right) \tag{18}$$

The precision of $\hat{\mu}_4$ attained optimality when $\alpha=1-2C_{YX}/C_X^2$, its bias and the $MSE(\hat{\mu}_4)_{\min}$ is given by

$$Bais(\hat{\mu}_4)=(1-\nu)\psi_{r,N}\bar{Y}\left(\frac{3}{8}C_X^2-\frac{1}{2}C_{YX}\right) \tag{19}$$

$$MSE(\hat{\mu}_4)_{\min}=\psi_{r,N}S_Y^2\left(1-\rho^2\right) \tag{20}$$

Where $\nu=1-2C_{YX}/C_X^2$

[33] proposed ratio exponential imputation scheme given in (21) to address the problem of compromised in [29]

$$y_{.i}=\begin{cases}y_i & if\ i\in\Phi\\ \dfrac{\bar{y}_r}{n-r}\left[n\eta\exp\left(\dfrac{\bar{X}-\bar{x}_r}{\bar{X}+\bar{x}_r+2\rho/\beta_2(x)}\right)-r\right] & if\ i\in\Phi^C\end{cases} \tag{21}$$

Under this method, the resultant estimator of the population mean $\bar{Y}$ given as

$$\hat{\mu}_5=\eta\hat{t}_0\exp\left(\left(\bar{X}-\bar{x}_r\right)\left(\bar{X}+\bar{x}+2\rho/\beta_2(x)\right)^{-1}\right) \tag{22}$$

$\hat{\mu}_5$ attained highest precision when $\eta=\bar{Y}^2/\left(\bar{Y}^2+\psi_{r,N}\left(S_y^2+0.25\vartheta^2R^2S_X^2-\vartheta RS_{YX}\right)\right)$, $\vartheta=\beta_2(x)\bar{X}/\left(\beta_2(x)\bar{X}+\rho_{YX}\right)$, its bias and the $MSE(\hat{\mu}_5)_{\min}$ is given by

$$Bais(\hat{\mu}_5)=\left[(\eta-1)+\frac{\vartheta}{8_r}\left(3\vartheta C_X^2-4\rho C_yC_x\right)\lambda\right]\bar{Y} \tag{23}$$

$$MSE(\hat{\mu}_5)_{\min}=\frac{\bar{Y}^2\left(\psi_{r,N}\left(S_Y^2+0.25\vartheta^2R^2S_X^2-\vartheta RS_{YX}\right)\right)}{\left(\bar{Y}^2+\psi_{r,N}\left(S_Y^2+0.25\vartheta^2R^2S_X^2-\vartheta RS_{YX}\right)\right)} \tag{24}$$

[31] proposed imputation scheme which is applicable when X and Y are positively or negatively correlated, for population mean estimators using linear combination approach

$$y_{.i}=\begin{cases}\alpha\dfrac{n}{r}y_i\dfrac{\bar{X}}{\bar{x}}+(1-\alpha)\bar{y}_r\dfrac{\bar{x}_r}{\bar{X}} & i\in\Phi\\ (1-\alpha)\bar{y}_r\dfrac{\bar{x}_r}{\bar{X}} & i\in\Phi^C\end{cases} \tag{25}$$

The point estimator of population mean $\bar{Y}$ under proposed method of imputation is:

$$\hat{\mu}_6=\bar{y}_t\left\{\alpha\frac{\bar{X}}{\bar{x}}+(1-\alpha)\frac{\bar{x}_r}{\bar{X}}\right\} \tag{26}$$

Where α is an unknown parameter to be estimated.

The bias, mean square error and minimum mean square error are given by:

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$$Bias\left(\hat{\mu}_6\right)=\bar{Y}\psi_{n,N}\left\{\alpha+\left(1-2\alpha\right)S_{YX}\right\}C_X^2 \quad (27)$$

$$MSE\left(\hat{\mu}_6\right)=\bar{Y}^2\begin{bmatrix}\psi_{n,N}C_Y^2+\psi_{n,N}\left(1+4\alpha^2-4\alpha\right)C_X^2\\ 2\left(1-2\alpha\right)\psi_{n,N}S_{YX}C_X^2\end{bmatrix} \quad (28)$$

$$MSE\left(\hat{\mu}_6\right)_{\min}=\bar{Y}^2\left(\psi_{n,N}-\psi_{n,N}\rho^2\right)C_Y^2 \quad (29)$$

[35] proposed the following generalized class of imputation schemes given in (30) with known parameters of auxiliary variable which does not depends on unknown parameters of study variable.

$$y_{.i}=\begin{cases}y_i & \mathrm{i}\in\Phi\\ \dfrac{\hat{\mu}_0+\hat{\beta}_{rg}\left(\bar{X}-\bar{x}_r\right)}{\pi_1\bar{x}_r+\pi_2}\left(\pi_1\bar{X}+\pi_2\right)\\ \exp\left(\dfrac{\varpi_1\left(\bar{X}-\bar{x}_r\right)}{\varpi_1\left(\bar{X}+\bar{x}_r\right)+2\varpi_2}\right) & \mathrm{i}\in\Phi^c\end{cases} \quad (30)$$

where π_1 and π_2 are known functions of auxiliary variables like coefficients of skewness $\beta_1\left(x\right)$, kurtosis $\beta_2\left(x\right)$, variation C_X, standard deviation S_X etc,

The point estimator of population mean $\bar{Y}$ under proposed scheme of imputation is

$$\hat{\mu}_8=\frac{r}{n}\hat{\mu}_0+\left(1-\frac{r}{n}\right)\frac{\hat{\mu}_0+\hat{\beta}_{rg}\left(\bar{X}-\bar{x}_r\right)}{\pi_1\bar{x}_r+\pi_2}$$
$$\left(\pi_1\bar{X}+\pi_2\right)\exp\left(\frac{\varpi_1\left(\bar{X}-\bar{x}_r\right)}{\varpi_1\left(\bar{X}+\bar{x}_r\right)+2\varpi_2}\right) \quad (31)$$

The bias and mean square error are given by

$$Bias\left(\hat{\mu}_8\right)=\psi_{r,N}\left(1-\frac{r}{n}\right)\left(\begin{matrix}\left(\begin{matrix}\beta_{rg}\bar{X}\left(\eta_1+\eta_2\right)+\\ \bar{Y}\left(\eta_1^2+\eta_1\eta_2-1.5\eta_2^2\right)\end{matrix}\right)\\ S_X^2-\bar{Y}\left(\eta_1+\eta_2\right)C_{YX}\end{matrix}\right) \quad (32)$$

$$MSE\left(\hat{\mu}_8\right)=\psi_{r,N}\left(S_Y^2+\Upsilon^2S_X^2-2\Upsilon S_{YX}\right) \quad (33)$$

where $\Upsilon=\left(1-\dfrac{r}{n}\right)\left(R\left(\eta_1+\eta_2\right)+\beta_{rg}\right)$

2. Materials and Methods

2.1 Proposed Imputation Schemes

Having studied the work of [35], the following imputation scheme is proposed.

$$y_{.i}=\begin{cases}y_i & \mathrm{i}\in\Phi\\ \left(\bar{y}_r+\hat{\beta}_{rg}\left(\bar{x}_n-\bar{x}_r\right)\right)\dfrac{\bar{x}_n}{\bar{x}_r}\exp\left(\dfrac{\tau_1\left(\bar{x}_n-\bar{x}_r\right)}{\tau_1\left(\bar{x}_n+\bar{x}_r\right)+2\tau_2}\right) & \mathrm{i}\in\Phi^c\end{cases} \quad (35)$$

where τ_1 and τ_2 are known functions of auxiliary variables like coefficients of skewness $\beta_1\left(x\right)$, kurtosis $\beta_2\left(x\right)$, variation C_X, standard deviation S_X, f_i, $i=1,2,3$.

2.2 Data for Empirical Study

In this section, simulation study was conducted to examine the superiority of the proposed estimators over other estimators considered in the study. Data of size 10000 units were generated for study population using function defined in Table 1 below. Samples of sizes 500 units from which 60 units were selected as respondents were randomly chosen 10,000 times by method SRSWOR. The efficiency (MSEs) and percentage gain in efficiency of the considered estimators were computed using (36) and (37) respectively.

$$MSE\left(\hat{\theta}_d\right)=\frac{1}{10000}\sum_{d=1}^{10000}\left(\hat{\theta}_d-\bar{Y}\right)^2 \quad (36)$$

$$PRE\left(\theta_l\right)=\left(\frac{MSE\left(\hat{\mu}_0\right)}{MSE\left(\theta\right)}\right)\times100 \quad (37)$$

Table 1: Simulation data used for empirical study

Pop	Study Variable	Aux. variable
1	$Y=3+0.4X+\varepsilon$	$X\sim unif(0.5,3)$
II	Where	$X\sim Norm(5,9)$
III	$\varepsilon\sim N\left(0,1\right)$	$X\sim chiq\left(1\right)$

3. Results and Discussion

3.1 Estimator and MSEs of Imputation Scheme under cases I and II

The estimator of the proposed scheme can be obtained using (3.1).

$$\hat{t}=\frac{1}{n}\left(\sum_{i\in\Phi}y_{.i}+\sum_{i\in\Phi^c}y_{.i}\right) \quad (38)$$

$$\hat{t}=\frac{1}{n}\left(\sum_{i=1}^{r}y_i+\sum_{i=1}^{n-r}\left(\begin{matrix}\left(\bar{y}_r+\hat{\beta}_{rg}\left(\bar{x}_n-\bar{x}_r\right)\right)\dfrac{\bar{x}_n}{\bar{x}_r}\\ \exp\left(\dfrac{\tau_1\left(\bar{x}_n-\bar{x}_r\right)}{\tau_1\left(\bar{x}_n+\bar{x}_r\right)+2\tau_2}\right)\end{matrix}\right)\right) \quad (39)$$

Therefore, the estimator of the proposed scheme is given as;

$$\hat{t}=\frac{r}{n}\bar{y}_r+\left(1-\frac{r}{n}\right)\exp\left(\frac{\tau_1\left(\bar{x}_n-\bar{x}_r\right)}{\tau_1\left(\bar{x}_n+\bar{x}_r\right)+2\tau_2}\right)$$
$$\left(v\left(\bar{y}_r+\hat{\beta}_{rg}\left(\bar{x}_n-\bar{x}_r\right)\right)+\left(1-v\right)\bar{y}_r\frac{\bar{x}_n}{\bar{x}_r}\right) \quad (40)$$

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The Mean Square Errors of $\left(\hat{t}\right)$ under Case I is defined as;

$$MSE\left(\hat{t}\right)_I = \Delta\Sigma\Delta' \quad (41)$$

where, $\Delta = \begin{pmatrix} \frac{\partial \hat{t}}{\partial \bar{y}_r} & \frac{\partial \hat{t}}{\partial \bar{x}_n} & \frac{\partial \hat{t}}{\partial \bar{x}_r} \end{pmatrix}$ is a matrix of order 1×3, Δ' is its transpose and the variance-covariance matrix is defined as

$$\Sigma = \begin{pmatrix} \psi_{r,N}S_y^2 & \psi_{n,N}S_{yx} & \psi_{r,N}S_{yx} \\ \psi_{n,N}S_{yx} & \psi_{n,N}S_x^2 & \psi_{n,N}S_x^2 \\ \psi_{r,N}S_{yx} & \psi_{n,N}S_x^2 & \psi_{r,N}S_x^2 \end{pmatrix}$$

is a 3×3 non-singular matrix.

The Mean Square Errors of $\left(\hat{t}\right)$ under Case I is derived by differentiate (3.4) by differentiating $\hat{t}$ with respect to $\bar{y}_r, \bar{x}_n, \bar{x}_r$ to get (42), (43) and (44) respectively

$$\frac{\partial \hat{t}}{\partial \bar{y}_r} = \frac{r}{n} + \left(1-\frac{r}{n}\right)\left(v + (1-v)\frac{\bar{x}_n}{\bar{x}_r}\right)\exp\left(\frac{\tau_1\left(\bar{x}_n - \bar{x}_r\right)}{\tau_1\left(\bar{x}_n + \bar{x}_r\right) + 2\tau_2}\right) \quad (42)$$

Let $H = \exp\left(\frac{\tau_1\left(\bar{x}_n - \bar{x}_r\right)}{\tau_1\left(\bar{x}_n + \bar{x}_r\right) + 2\tau_2}\right)$, then

$$\frac{\partial H}{\partial \bar{x}_r} = -\left(\frac{\left(\tau_1\left(\bar{x}_n + \bar{x}_r\right) + 2\tau_2\right)\tau_1 + \tau_1\left(\bar{x}_n - \bar{x}_r\right)}{\left(\tau_1\left(\bar{x}_n + \bar{x}_r\right) + 2\tau_2\right)^2}\right)H \quad (43)$$

$$\frac{\partial H}{\partial \bar{x}_n} = \frac{\left(\left(\tau_1\left(\bar{x}_n + \bar{x}_r\right) + 2\tau_2\right)\tau_1 - \tau_1\left(\bar{x}_n - \bar{x}_r\right)\right)}{\left(\tau_1\left(\bar{x}_n + \bar{x}_r\right) + 2\tau_2\right)^2}H \quad (44)$$

As $r \to N, n \to N$,

then $\bar{x}_n = \bar{X}$, $\bar{x}_r = \bar{X}$, $\hat{\mu}_0 = \bar{Y}, \hat{\beta}_{rg} = \beta_{rg}$ in (42), (43), and (44) we have,

$$\frac{\partial \hat{t}}{\partial \bar{y}_r} = \frac{r}{n} + \left(1 - \frac{r}{n}\right) = 1 \quad (45)$$

$$\frac{\partial \hat{t}}{\partial \bar{x}_r} = -\theta \quad (46)$$

$$\frac{\partial \hat{t}}{\partial \bar{x}_n} = \theta \quad (47)$$

where

$$\theta = \left(1-\frac{r}{n}\right)\left(\frac{\bar{Y}\tau_1}{2\left(\tau_1\bar{X} + \tau_2\right)} + \frac{\bar{Y}}{\bar{X}} + \left(\beta_{rg} - \frac{\bar{Y}}{\bar{X}}\right)v\right) \quad (48)$$

Substituting (45), (46) and (47) into (41) to obtain the MSE of estimator $\hat{t}$ under case one as

$$MSE\left(\hat{t}\right)_I = \bar{Y}^2\begin{pmatrix} \psi_{r,N}C_y^2 - 2\theta\frac{\bar{X}}{\bar{Y}} \\ \left(\psi_{r,N} - \psi_{n,N}\right)\rho C_y C_x + \\ \theta^2\frac{\bar{X}^2}{\bar{Y}^2}\left(\psi_{r,N} - \psi_{n,N}\right)C_x^2 \end{pmatrix} \quad (49)$$

Differentiating (3.12) with respect to θ, we obtain,

$$\theta = \frac{\bar{Y}\rho C_y}{\bar{X}C_x} \quad (50)$$

Equate (48) and (50) and solve for v,

$$v_{opt(I)} = \frac{1}{\left(\beta_{rg} - \frac{\bar{Y}}{\bar{X}}\right)}\left(\frac{\bar{Y}\rho C_y}{\bar{X}C_x\left(1-\frac{r}{n}\right)} - \frac{\bar{Y}\tau_1}{2\left(\tau_1\bar{X} + \tau_2\right)} - \frac{\bar{Y}}{\bar{X}}\right) \quad (51)$$

Substituting (3.14) into (3.11) to obtain the minimum MSE of the estimator $\left(t_2\right)_I$ as

$$MSE\left(\hat{t}\right)_{I\min} = \bar{Y}^2C_y^2\left(\psi_{r,N} - \left(\psi_{r,N} - \psi_{n,N}\right)\rho^2\right) \quad (52)$$

The Mean Square Errors of $\left(\hat{t}\right)$ under Case II is derived using as;

$$MSE\left(\hat{t}_i\right)_{II} = \Delta\Sigma^*\Delta', \; i = 1,2,3,4 \quad (53)$$

where, $\Sigma^* = \begin{pmatrix} \psi_{r,N}S_y^2 & 0 & \psi_{r,N}S_{yx} \\ 0 & \psi_{n,N}S_x^2 & 0 \\ \psi_{r,N}S_{yx} & 0 & \psi_{r,N}S_x^2 \end{pmatrix}$

is a 3×3 non-singular matrix.

Substituting (45), (46), (47) into equation (53) to obtain the MSE of $\hat{t}_2$ under case two as

$$MSE\left(\hat{t}\right)_{II} = \bar{Y}^2\begin{pmatrix} \psi_{r,N}C_y^2 - 2\theta\frac{\bar{X}}{\bar{Y}}\psi_{r,N}\rho C_y C_x \\ +\theta_4^2\frac{\bar{X}^2}{\bar{Y}^2}\left(\psi_{n,N} + \psi_{r,N}\right)C_x^2 \end{pmatrix} \quad (54)$$

Differentiating (54) with respect to θ, we have,

$$\theta = \frac{\psi_{r,N}\rho\bar{Y}C_y}{\left(\psi_{n,N} + \psi_{r,N}\right)\bar{X}C_x} \quad (55)$$

Equate (48) and (55) and solve for v,

$$v_{opt(II)} = \frac{1}{\left(\beta_{rg} - \frac{\bar{Y}}{\bar{X}}\right)}\begin{pmatrix} \frac{\bar{Y}\lambda\rho C_y}{\left(\psi_{n,N} + \psi_{r,N}\right)\bar{X}C_x\left(1-\frac{r}{n}\right)} - \\ \frac{\bar{Y}\tau_1}{2\left(\tau_1\bar{X} + \tau_2\right)} - \frac{\bar{Y}}{\bar{X}} \end{pmatrix} \quad (56)$$

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Substituting (56) into (54), the minimum MSE of the estimator $\hat{t}$ is obtained as

$$MSE\left(\hat{t}\right)_{II\,\min} = \bar{Y}^2 C_y^2\left(\psi_{r,N} - \frac{{\psi_{r,N}}^2\rho^2}{\left(\psi_{n,N}+\psi_{r,N}\right)}\right) \quad (57)$$

3.2 Theoretical Efficiency Comparison

In this section, efficiency conditions of the proposed estimator over sample mean $\hat{\mu}_0$, and [35] $\hat{\mu}_8$ were established.

i. Sample mean Vs Proposed Estimator

$$Var\left(\hat{\mu}_0\right) - MSE\left(\hat{t}\right)_I > 0 \;\Rightarrow\; |\rho| > \sqrt{\frac{\psi_{r,N}-1}{\psi_{r,N}-\psi_{n,N}}} \quad (58)$$

$$Var\left(\hat{\mu}_0\right) - MSE\left(\hat{t}\right)_{II} > 0 \quad \Rightarrow \quad |\rho| > 0 \quad (59)$$

ii. [35] Vs Proposed Estimators

$$MSE\left(\hat{\mu}_8\right) - MSE\left(\hat{t}\right)_I > 0$$

$$\Rightarrow \; |\rho| > \frac{\Upsilon S_x\left(\sqrt{\psi_{r,N}\psi_{n,N}} + \psi_{r,N}\right)}{S_y\left(\psi_{r,N}-\psi_{n,N}\right)} \quad (60)$$

$$MSE\left(\hat{\mu}_8\right) - MSE\left(\hat{t}\right)_{II} > 0$$

$$\Rightarrow |\rho| = \frac{\Upsilon S_x\left(\sqrt{\psi_{n,N}\left(\psi_{n,N}+\psi_{r,N}\right)} + \psi_{n,N} + \psi_{r,N}\right)}{S_y \psi_{r,N}} \quad (61)$$

3.3 Numerical Efficiency Comparisons

In this subsection, numerical results on the efficiency of the proposed imputation schemes with respect to the existing imputation schemes is presented.

Table 2: MSE and PRE of Proposed and Other Estimators using Population I

Estimators	**MSE**	**PRE**	**Estimators**	**MSE**	**PRE**
Mean $\left(\hat{\mu}_0\right)$	89.189	100.00	Estimators of [35]		
[40] $\left(\hat{\mu}_1\right)$	50.501	176.61			
[21] $\left(\hat{\mu}_2\right)$	44.883	198.71	$\left(\hat{\mu}_8\right)_1$	35.7908	249.1954
[23] $\left(\hat{\mu}_3\right)$	78.210	114.04	$\left(\hat{\mu}_8\right)_2$	80.71733	110.4955
[29] $\left(\hat{\mu}_4\right)$	82.923	107.56	$\left(\hat{\mu}_8\right)_3$	79.24314	112.5511
[33] $\left(\hat{\mu}_5\right)$	77.445	115.17	$\left(\hat{\mu}_8\right)_4$	38.3721	232.4319
[31] $\left(\hat{\mu}_6\right)$	72.200	123.53	$\left(\hat{\mu}_8\right)_5$	68.40027	130.3928
			$\left(\hat{\mu}_8\right)_6$	71.82467	124.1760
			$\left(\hat{\mu}_8\right)_7$	26.1457	341.1231
Proposed Estimators					
Case I	**MSE**	**PRE**	$\left(\hat{\mu}_8\right)_8$	51.29447	173.8765
$\left(\hat{t}_{11}\right)_1$	14.2876	624.2408	$\left(\hat{\mu}_8\right)_9$	44.80066	199.0797
$\left(\hat{t}_{12}\right)_1$	22.34439	399.1562	$\left(\hat{\mu}_8\right)_{10}$	36.635	243.4530
$\left(\hat{t}_{13}\right)_1$	31.3448	284.5417	$\left(\hat{\mu}_8\right)_{11}$	57.68069	154.6254
$\left(\hat{t}_{14}\right)_1$	22.30547	399.8527	$\left(\hat{\mu}_8\right)_{12}$	33.67027	264.8895
Case II			$\left(\hat{\mu}_8\right)_{13}$	33.27638	268.0250
$\left(\hat{t}_{11}\right)_{II}$	1.193615	7472.1773	$\left(\hat{\mu}_8\right)_{14}$	30.9571	288.1052
$\left(\hat{t}_{12}\right)_{II}$	26.91738	331.3436	$\left(\hat{\mu}_8\right)_{15}$	52.52582	169.8003
$\left(\hat{t}_{13}\right)_{II}$	16.91738	527.2035	$\left(\hat{\mu}_8\right)_{16}$	54.09525	164.8740
$\left(\hat{t}_{14}\right)_{II}$	19.3615	460.6514	$\left(\hat{\mu}_8\right)_{17}$	39.5766	225.3579

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Table 3: MSE and PRE of Proposed and Other Estimators using Population II

Estimators	MSE	PRE	Estimators	MSE	PRE
Mean $(\hat{\mu}_0)$	107.824	100	Estimators of [35]		
[40] $(\hat{\mu}_1)$	60.74127	177.5136			
[21] $(\hat{\mu}_2)$	16.876	638.9192	$(\hat{\mu}_8)_1$	43.0572	250.4203
[23] $(\hat{\mu}_3)$	94.32676	114.309	$(\hat{\mu}_8)_2$	91.04322	118.4316
[29] $(\hat{\mu}_4)$	38.603	279.3151	$(\hat{\mu}_8)_3$	89.35996	120.6625
[33] $(\hat{\mu}_5)$	94.30049	114.3409	$(\hat{\mu}_8)_4$	134.4198	80.2143
[31] $(\hat{\mu}_6)$	52.576	205.078	$(\hat{\mu}_8)_5$	77.01392	140.0058
			$(\hat{\mu}_8)_6$	80.90649	133.2699
Proposed Estimators			$(\hat{\mu}_8)_7$	26.1457	341.1231
Case I	**MSE**	**PRE**	$(\hat{\mu}_8)_8$	68.95607	156.3662
$(\hat{t}_{11})_1$	24.923	432.613	$(\hat{\mu}_8)_9$	84.29426	127.9138
$(\hat{t}_{12})_1$	33.598	320.921	$(\hat{\mu}_8)_{10}$	132.4046	81.4352
$(\hat{t}_{13})_1$	33.635	320.564	$(\hat{\mu}_8)_{11}$	70.52595	152.8855
$(\hat{t}_{14})_1$	24.642	437.552	$(\hat{\mu}_8)_{12}$	37.89331	284.5462
Case II			$(\hat{\mu}_8)_{13}$	99.45384	108.4161
$(\hat{t}_{11})_{II}$	19.875	542.484	$(\hat{\mu}_8)_{14}$	64.86881	166.2185
$(\hat{t}_{12})_{II}$	22.005	489.979	$(\hat{\mu}_8)_{15}$	96.54145	111.6867
$(\hat{t}_{13})_{II}$	22.005	489.979	$(\hat{\mu}_8)_{16}$	94.90442	113.6132
$(\hat{t}_{14})_{II}$	19.875	542.494	$(\hat{\mu}_8)_{17}$	94.9044	113.6133

Table 4: MSE and PRE of Proposed and Other Estimators using Population III

Estimators	MSE	PRE	Estimators	MSE	PRE
Mean $(\hat{\mu}_0)$	28865	100.0000	Estimators of [35]		
[40] $(\hat{\mu}_1)$	26818	107.6329			
[21] $(\hat{\mu}_2)$	18010	160.2721	$(\hat{\mu}_8)_1$	10211	282.6853
[23] $(\hat{\mu}_3)$	24993	115.4923	$(\hat{\mu}_8)_2$	9002	320.6510
[29] $(\hat{\mu}_4)$	18010	160.2721	$(\hat{\mu}_8)_3$	10001	288.6211
[33] $(\hat{\mu}_5)$	24993	115.4923	$(\hat{\mu}_8)_4$	8791	328.3472
[31] $(\hat{\mu}_6)$	15635	184.617	$(\hat{\mu}_8)_5$	9219	313.1034
			$(\hat{\mu}_8)_6$	9732	296.5988
Proposed Estimators			$(\hat{\mu}_8)_7$	26.1457	341.1231
Case I	**MSE**	**PRE**	$(\hat{\mu}_8)_8$	8711	331.3626
$(\hat{t}_{11})_1$	4867	593.075	$(\hat{\mu}_8)_9$	8681	332.5078
$(\hat{t}_{12})_1$	4454	648.069	$(\hat{\mu}_8)_{10}$	9172	314.7078
$(\hat{t}_{13})_1$	3448	837.152	$(\hat{\mu}_8)_{11}$	9812	294.1806

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$(\hat{t}_{14})_1$	5111	564.762	$(\hat{\mu}_8)_{12}$	8548	337.6813
Case II	**MSE**	**PRE**	$(\hat{\mu}_8)_{13}$	7638	377.9131
$(\hat{t}_{11})_{II}$	2615	1103.824	$(\hat{\mu}_8)_{14}$	9571	301.5881
$(\hat{t}_{12})_{II}$	3728	774.275	$(\hat{\mu}_8)_{15}$	9267	311.4816
$(\hat{t}_{13})_{II}$	3738	772.204	$(\hat{\mu}_8)_{16}$	9525	303.0446
$(\hat{t}_{14})_{II}$	3615	798.478	$(\hat{\mu}_8)_{17}$	9766	295.5663

Tables 2, 3 and 4 show MSEs and PREs of the proposed and some existing related estimators using model I, II and III respectively from the simulation in table 1. The result revealed the MSEs and PREs of the proposed estimators for estimating missing value using two-phase sampling technique and other related estimators considered in the study. The results show that all the proposed estimators have minimum MSE and higher PRE compared to the conventional estimators and other imputation techniques considered in this study. This implies that the proposed class of estimators are more efficient in estimation of missing values than other related estimators considered in this research.

4. Conclusion

By considering the results obtained from the empirical study on the efficiency of the proposed estimators over some exists related estimators considered in the study. Therefore, it is concluded that the proposed estimators have minimum MSE compared to other estimators considered in all the numerical computations carried out in the study, hence, the proposed estimators demonstrated high level of efficiency over other estimator considered in this study and therefore, minimizes resources for collecting information in mail survey characterized with non-response.

Conflict of Interest

The author declares that there is no conflict of interest.

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